Sampling and A/D conversion

= 2 step procedure

• *Sampling or discretization* — Taking discrete samples of a continuous data stream. The data may still be in analog representation after the sampling process.

• *Analog to digital conversion (quantization and coding)* — For voltage signals, this steps normally occurs in an electronic device which is called ADC, 'analog to digital converter'.

The sampling process

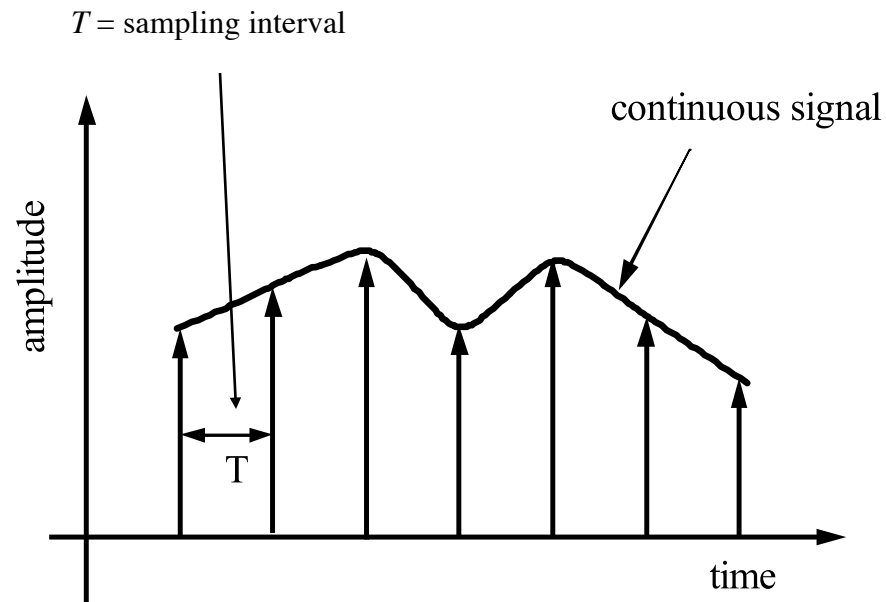$1/T = f_{dig}$, is called the *sampling frequency* or the *digitization frequency*.

T = sampling interval

continuous signal

T

amplitude

time

Fig. 5.1 Sketch of the discretization (sampling) process. The vertical arrows show the locations and the values of the samples. T denotes the sampling interval.

I) Discrete Signals

1.Discretization - Sampling

A continuous Signal Function:$x_a(t)$ taken at specific time steps T_s results in:

$$x[n] = x_a(nT_s);$$

T_s = sampling interval; $f_s = \dfrac{1}{T_s}$ = sampling rate or sampling frequency

⚠ Note! The amplitude values are still $x_a(t) \in \mathbf{R}$!!!!

The Sampling Theorem
In order to describe a continuous signal or function complete and unique using amplitude values taken at discrete times T_s, the sampled signal **MUST NOT**

HAVE energy above a certain frequency $\dfrac{f_s}{2} = \dfrac{1}{2T_s}$. This frequency is also called

Nyquist-Frequency.

The corresponding continuous signal $x_a(t)$ could be reconstructed using a linear

combination of the discrete function weighted by a function $\mathrm{sinc}(t) = \dfrac{\sin(t)}{t}$:

$$x_a(t) = \sum_{n=-\infty}^{\infty} x[n]\,\mathrm{sinc}(\pi f_s(t - nT_s))$$

⚠ The sampling theorem **MUST** be applied **BEFORE** the sampling process. Therefore an analog lowpass filter must be applied before sampling - regardless which sampling frequency is used. The corner frequency (!) of that filter should satisfy:

$$f_c = 0.4 \cdot f_s.$$

Consequence of violation: **ALIASING**
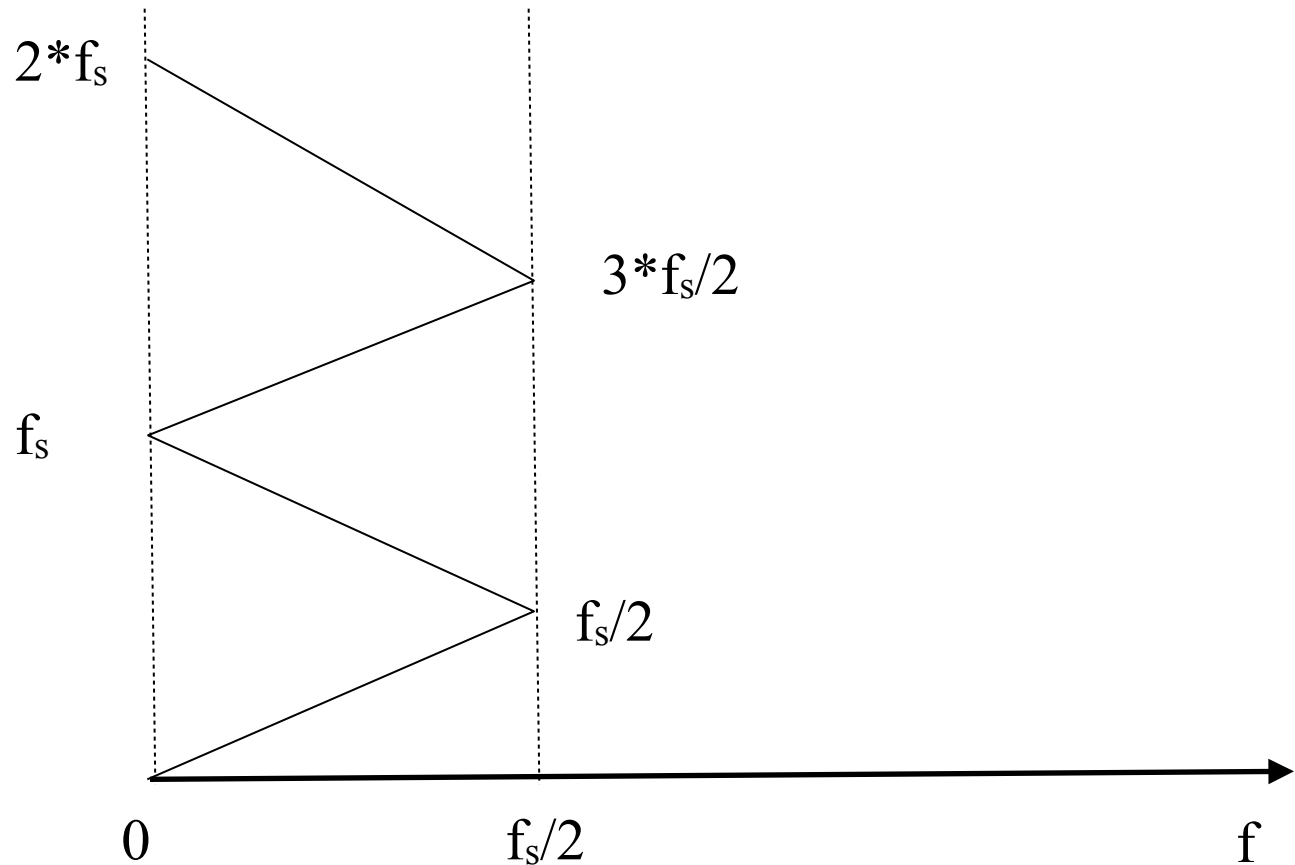
Problem: We are sampling a continuous process with the sampling rate of 125 Hz. Estimate the alias frequencies of signals at 70, 120 and 300 Hz
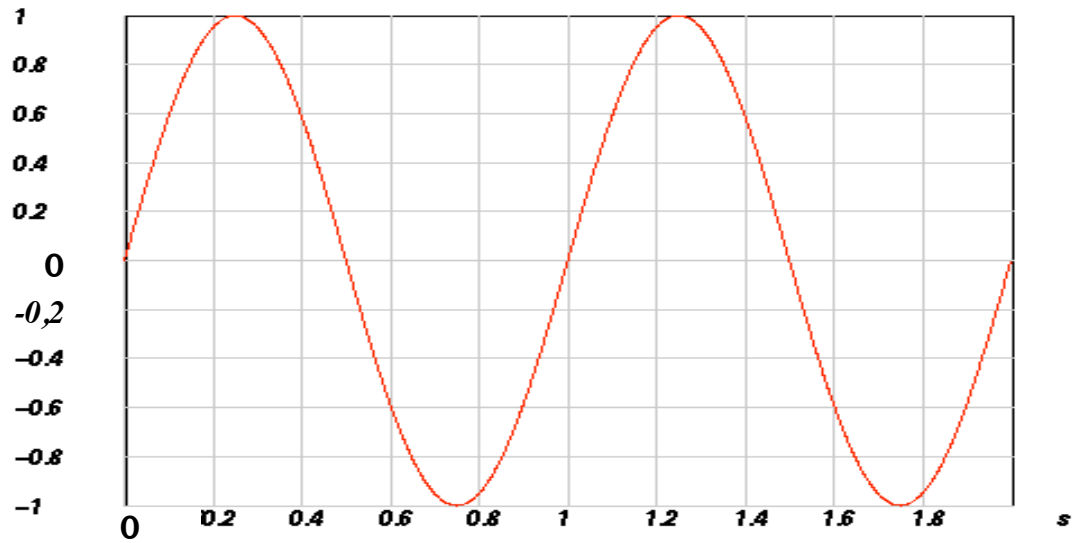
Fig. 5.2 Input signal for the simulation of the discretization process. The signal frequency is 1 Hz.
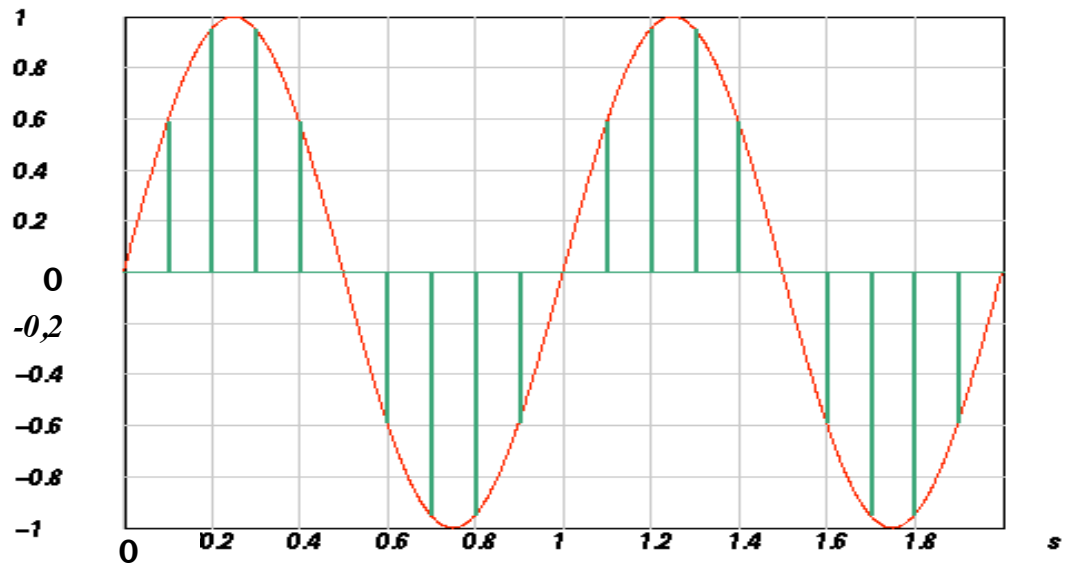
Fig. 5.3 Discretizing the data trace of Fig. 5.2 using a discretization frequency of 10 Hz. The vertical bars show the locations and the values of the function at the sampled times.

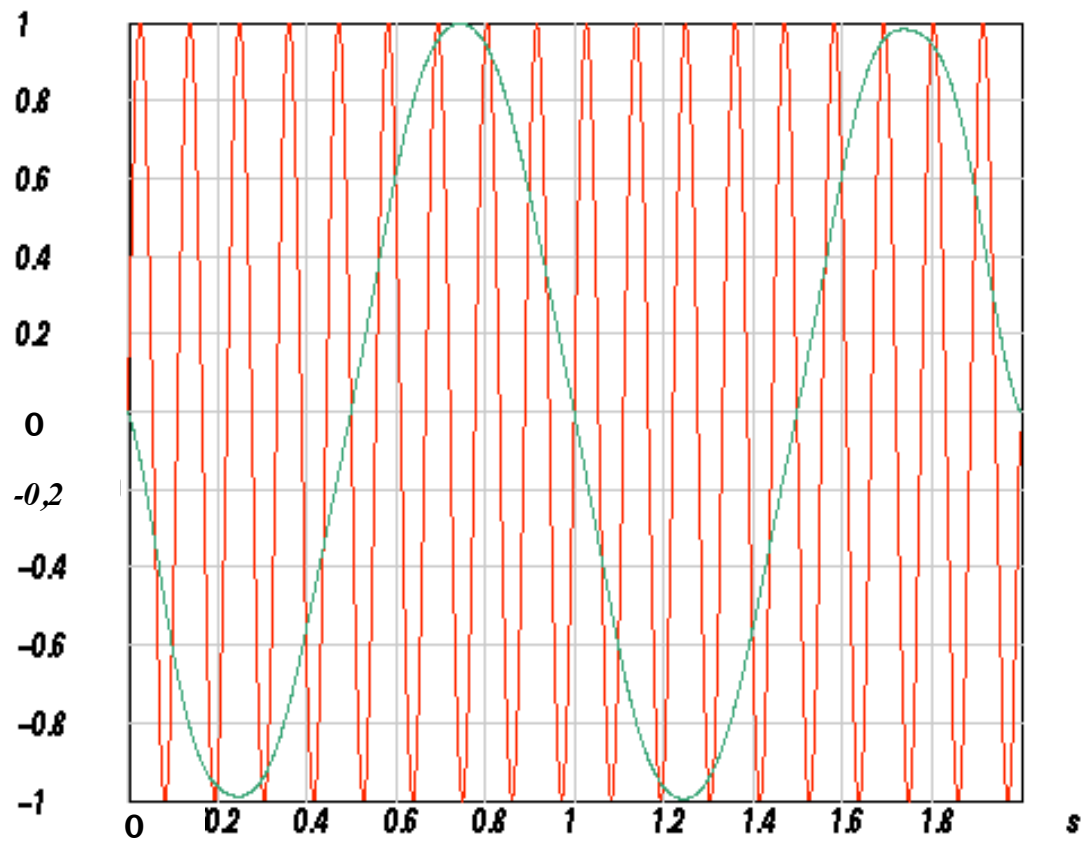**Fig. 5.4** Original and reconstructed trace of Fig. 5.2 (after discretizing all of them with 10 Hz prior to reconstruction).
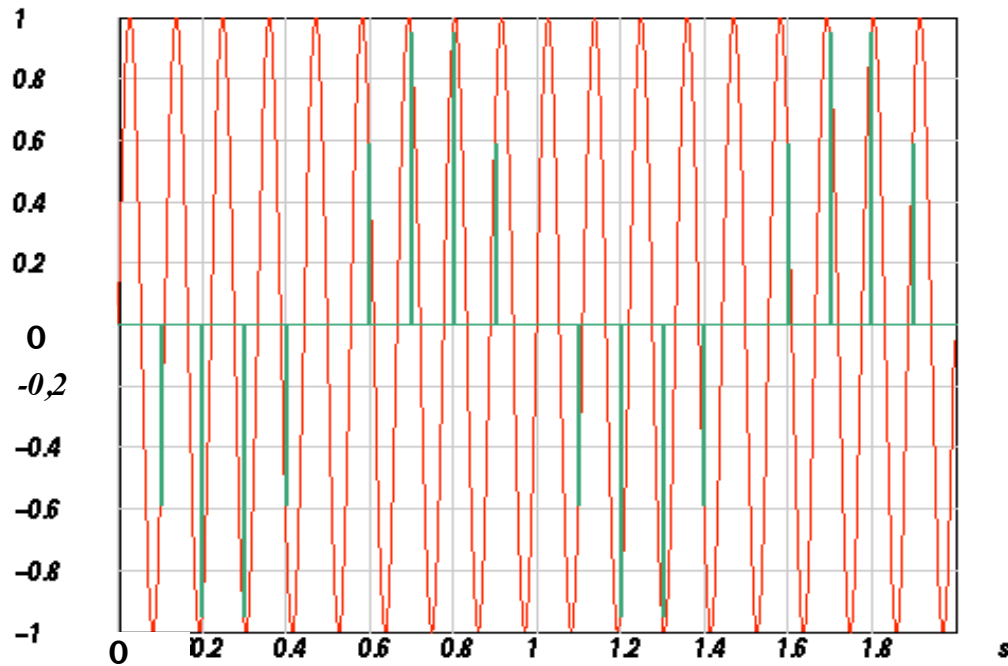
Fig. 5.6 Discretizing a sinusoidal signal with a signal frequency of 9 Hz and discretization frequency of 10 Hz. The vertical bars show the locations and the values of the function at the sampled times

Fig. 5.5 Original and reconstructed sinusoidal signal with a signal frequency of 9 Hz (discretization frequency 10 Hz).

3.A/D-Conversion

Decimal system:

$$x_{(10)} = \sum_i d_i^{(10)} 10^i;$$

Example:

$$1024_{(10)} = 4 \cdot 10^0 + 2 \cdot 10^1 + 0 \cdot 10^2 + 1 \cdot 10^3$$

            LSB   MSB

Binary system:

$$x_{(2)} = \sum_i d_i^{(2)} 2^i$$

Example:

$$512_{(10)} = 0 \cdot 2^0 + \ldots + 0 \cdot 2^8 + 1 \cdot 2^9 \text{ represents "Little Endian"}$$

            LSB   MSB   000000001

A 16 bit A/D-converter could represent in principle 2^{16} output states in its maximum (values between 0 - $(2^n - 1)$) are possible).

The LSB (least significant bit) or smallest step width of the A/D-converter (resolution) is defined by:

$$LSB = \frac{\text{Maximale Voltage}}{2^n} = Q.$$

As the resolution is directly dependent on the number of bits, a n-bit A/D-converter has "n-bit" resolution. Unfortunately, there is no rule, which would specify a "critical" number of "must have" bits. It is simply like that: if we have more bits we will decrease the noise added to the signal
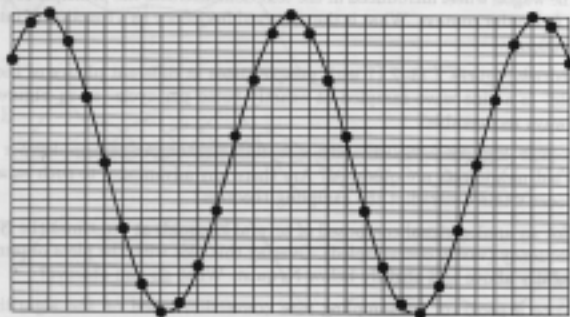
Figure 2.20: Conversion of an analog signal into a corresponding digital one involves quantizing both axes, sampling time and digitizing signal value. In the figure we see the original analog signal overlaid with the sampled time and digitized signal value grid. The resulting digital signal is depicted by the dots.
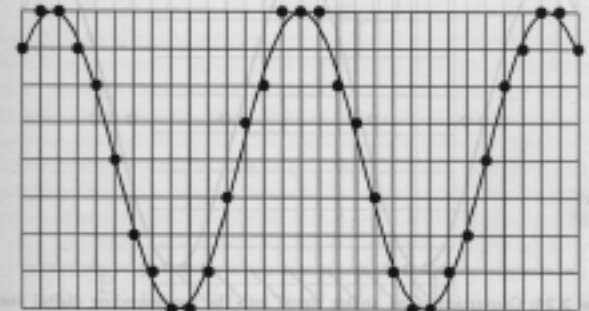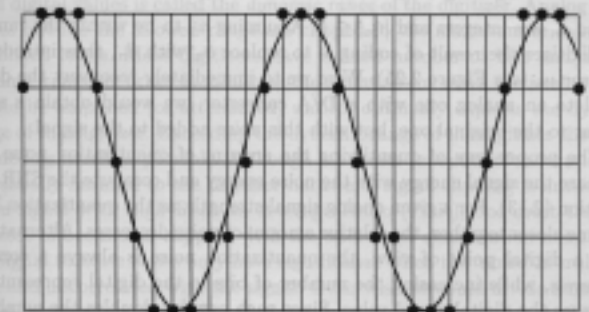
2.9 Digitization
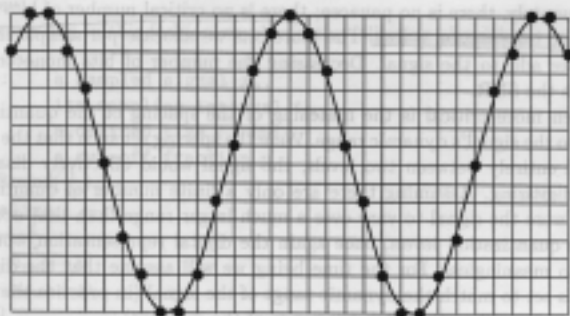
Figure 2.21: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. As in the previous figure the original analog signal has been overlaid with the sampled time and digitized signal value grid. However, here only 17 levels (about four bits) are used to represent the signal.

Figure 2.22: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. Once again the original analog signal has been overlaid with the sampled time and digitized signal value grid. Here only nine levels (a little more than three bits) are used to represent the signal.

Figure 2.23: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. Once again the original analog signal has been overlaid with the sampled time and digitized signal value grid. Here only five levels (about two bits) are used to represent the signal.

An equivalent important parameter of A/D conversion is the so called dynamic range:

$$D = 20\log_{10}\left(\frac{A_{max}}{A_{min}}\right)$$
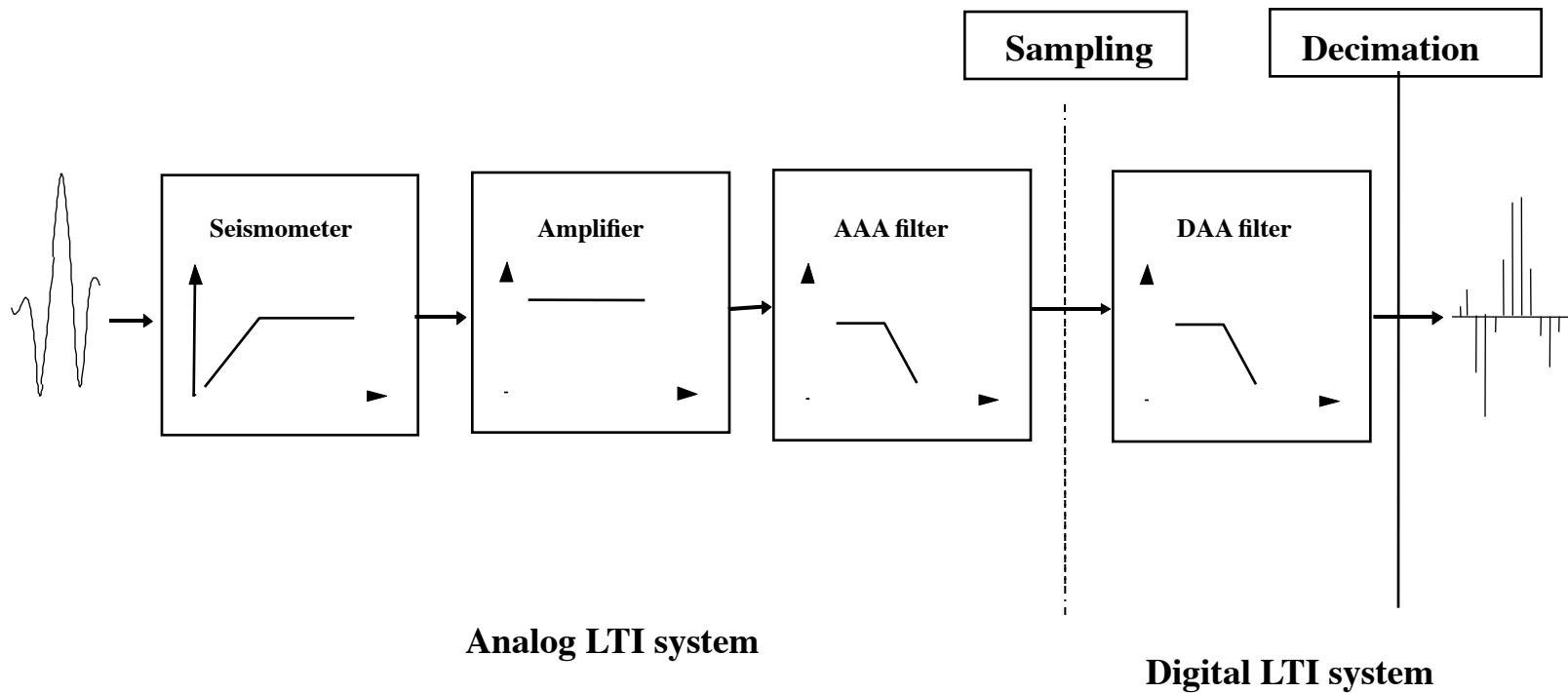
and therefore

$$D = 20\log_{10}(2^n - 1) \approx n\log_{10}(2) = n \cdot 6$$

16 bit A/D-converter: 90dB;
24 bit A/D-converter: 138 dB;

Be aware of the sign!

FIR - Filter Effects

Sampling

Decimation

| Seismometer | Amplifier | AAA filter | DAA filter |

Analog LTI system

Digital LTI system

Why bothering?

What is the reason for doing FIR filtering and decimating?

Nearly all seismic recorders use the oversampling technique to increase the resolution of recordings. In order to achieve an optimum valid frequency band, the filters are very steep.

Besides its advantages this also bears new problems.

```
#          << IRIS SEED Reader, Release 4.4 >>
#
#          ======== CHANNEL RESPONSE DATA ========
B050F03     Station:    RJOB
B050F16     Network:    BW
B052F03     Location:   ??
B052F04     Channel:    EHZ
B052F22     Start date: 2007,199
B052F23     End date:   No Ending Time
#          =======================================
#          +          +--------------------------------------------+          +
#          +          |  Response (Poles & Zeros),  RJOB ch EHZ  |          +
#          +          +--------------------------------------------+          +
#
B053F03     Transfer function type:              A [Laplace Transform (Rad/sec)]
B053F04     Stage sequence number:               1
B053F05     Response in units lookup:            M/S - Velocity in Meters per Second
B053F06     Response out units lookup:           V - Volts
B053F07     A0 normalization factor:          6.0077E+07
B053F08     Normalization frequency:            1
B053F09     Number of zeroes:                 2
B053F14     Number of poles:                  5
#          Complex zeroes:
#           i real       imag       real_error   imag_error
B053F10-13   0 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
B053F10-13   1 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
#          Complex poles:
#           i real       imag       real_error   imag_error
B053F15-18   0 -3.700400E-02 3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18   1 -3.700400E-02 -3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18   2 -2.513300E+02 0.000000E+00 0.000000E+00 0.000000E+00
B053F15-18   3 -1.310400E+02 -4.672900E+02 0.000000E+00 0.000000E+00
B053F15-18   4 -1.310400E+02 4.672900E+02 0.000000E+00 0.000000E+00
```

Seismometer
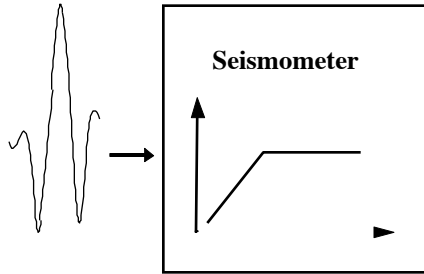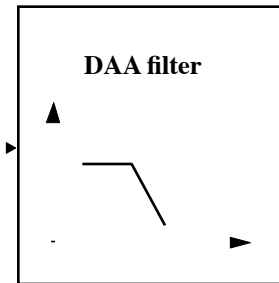
```
#          +              +-------------------------------+              +
#          +              |  FIR response,  RJOB ch EHZ  |              +
#          +              +-------------------------------+              +
#
B061F03    Stage sequence number:            3
B061F05    Symmetry type:            A
B061F06    Response in units lookup:          COUNTS - Digital Counts
B061F07    Response out units lookup:          COUNTS - Digital Counts
B061F08    Number of numerators:            96
#        Numerator coefficients:
#         i, coefficient
B061F09     0  3.767143E-09
B061F09     1  5.277283E-07
B061F09     2  2.184651E-06
B061F09     3 -5.639535E-06
B061F09     4 -1.233773E-06
B061F09     5  9.386712E-06
B061F09     6  4.859924E-06
B061F09     7 -1.644319E-05

…
#
#          +              +-----------------------------+              +
#          +              |  Decimation,  RJOB ch EHZ  |              +
#          +              +-----------------------------+              +
#
B057F03    Stage sequence number:            4
B057F04    Input sample rate:            1.000000E+03
B057F05    Decimation factor:            5
B057F06    Decimation offset:            0
B057F07    Estimated delay (seconds):          1.490000E-01
B057F08    Correction applied (seconds):        0.000000E+00
```

DAA filter

Linear Difference Equation

$$\sum_{k\,=\,0}^{N} a_k y[n-k] = \sum_{l\,=\,0}^{M} b_l x[n-l]$$

Infinite Impulse Response: $a_k \neq 0$

Finite Impulse Response: $a_0 = 1$; $a_{k \neq 0} = 0$

- **FIR filters** :

+   Always stable.
-    Steep filters need many coefficients.
+   Both causal and noncausal filters can be implemented.
+   Filters with given specifications are easy to implement!

- **IIR filters** :

-    Potentially unstable and subject to quantization errors.
+   Steep filters can easily be implemented with a few coefficients. Speed.
-    Filters with given specifications are in general, difficult, if not impossible, to implement *exactly(!).*

In SEED the impulse response of the decimation filters are given. But how to construct FIR filters?

Easiest way: inverse DFT with selected spectral shape and phase and truncate the (infinite) sequence to form a finite impulse response
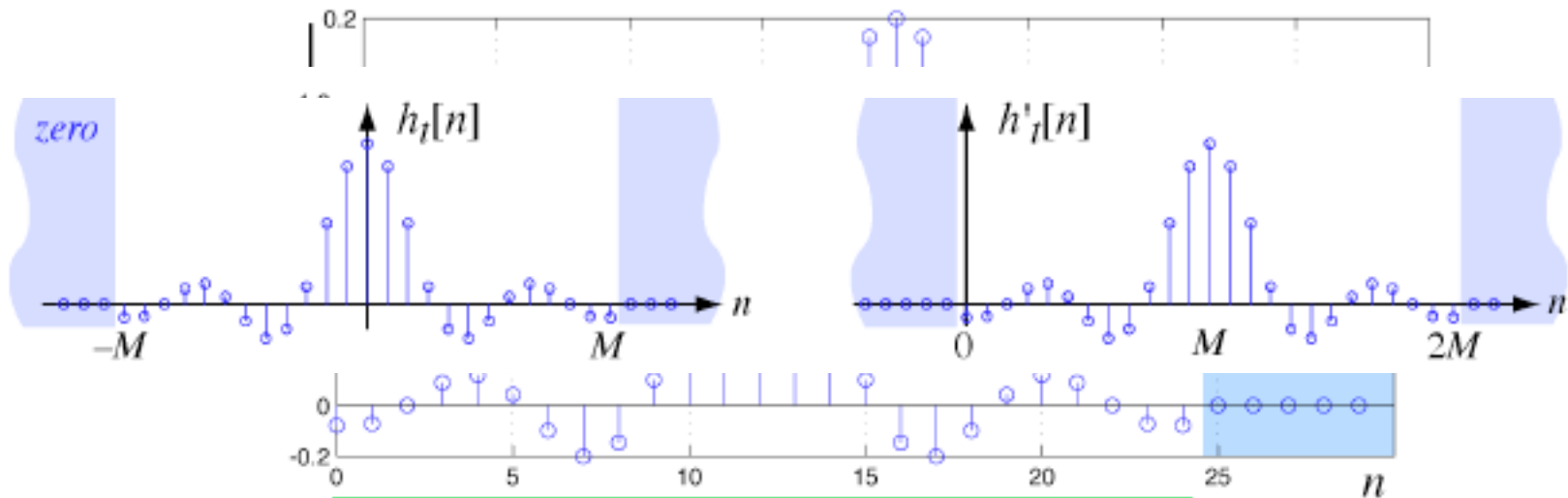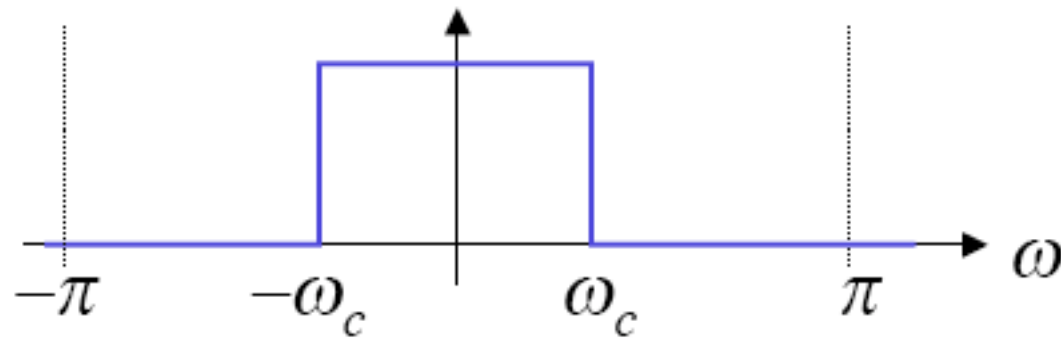
Filter length vs. Steepnes

$$H_d(e^{j\omega})$$

$$\circledast$$

$$\text{DTFT}\{w_R[n]\}$$
periodic sinc...
$$\frac{\sin Nx}{\sin x}$$

$$=$$

$$H_t(e^{j\omega})$$

Rectangular

big sidelobes!

Hanning

double width mainlobe

Hamming

reduced 1st sidelobe

Blackman

triple width mainlobe
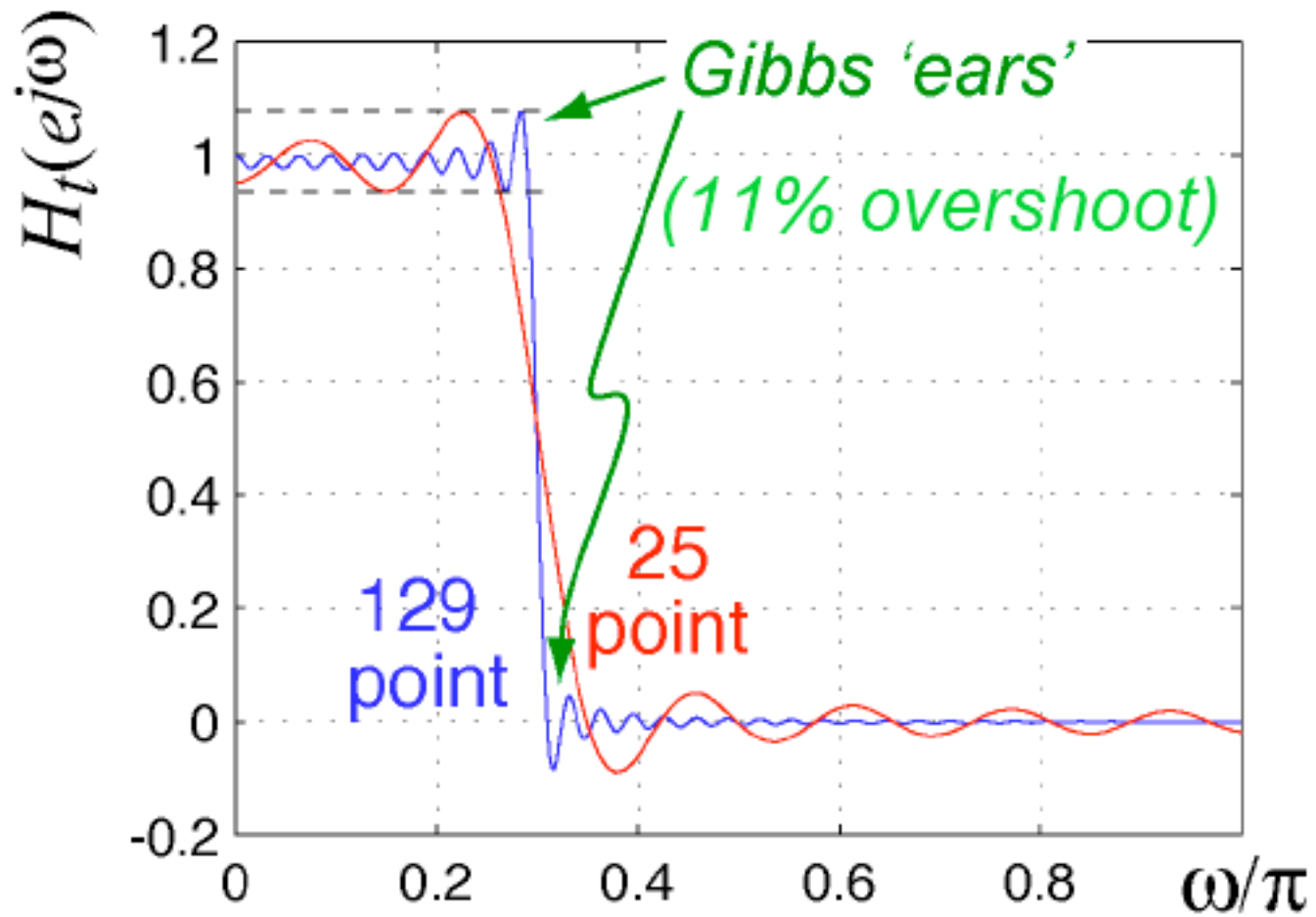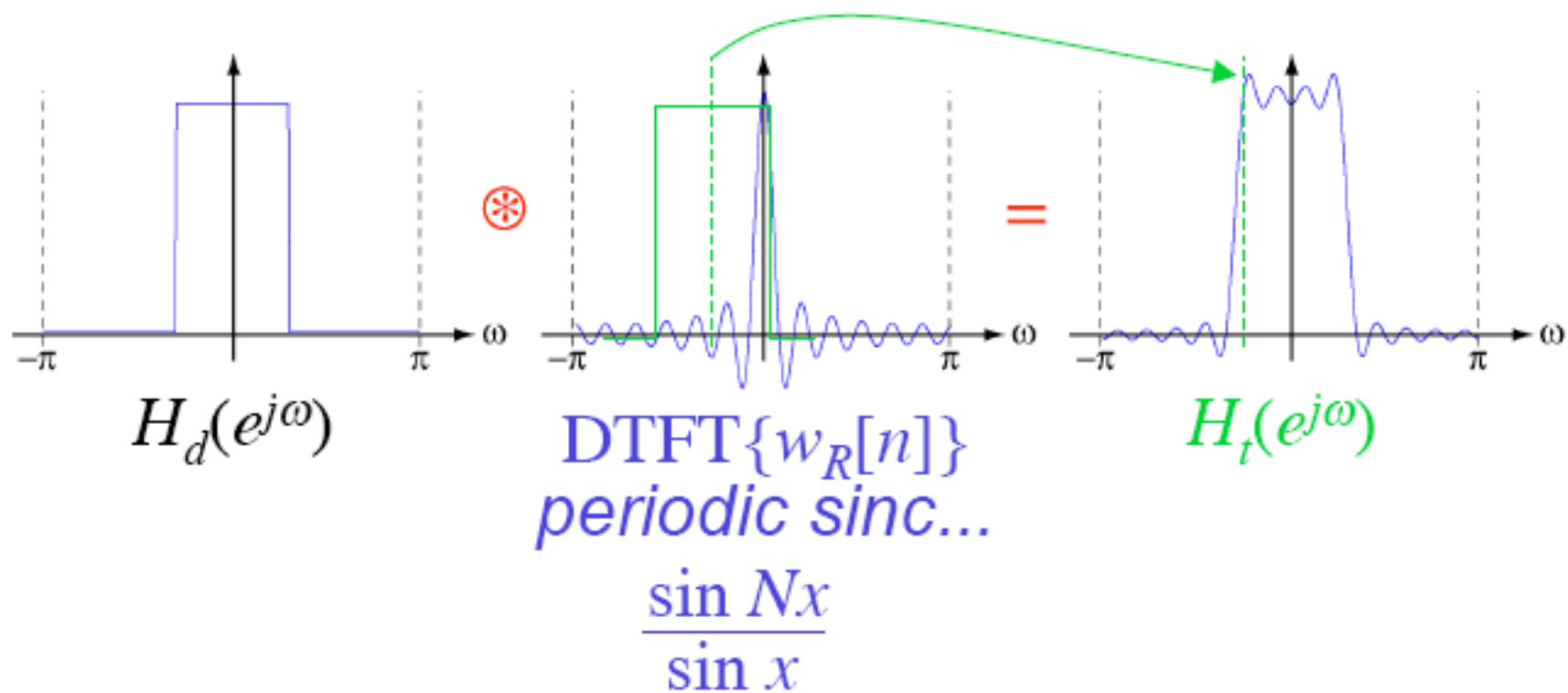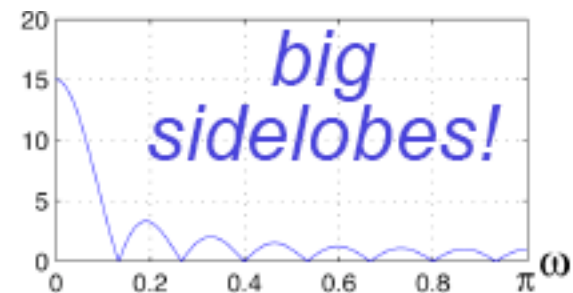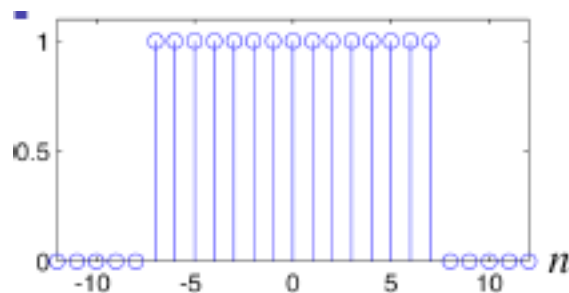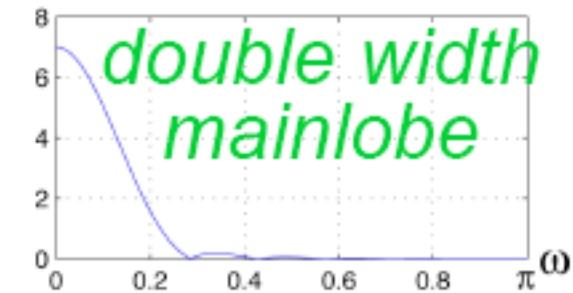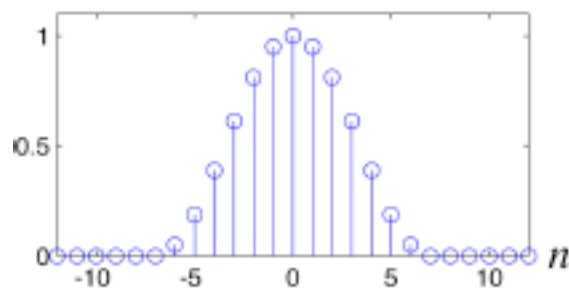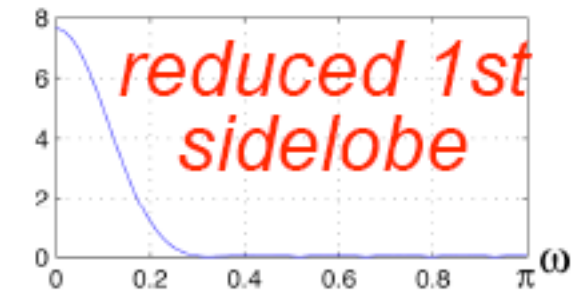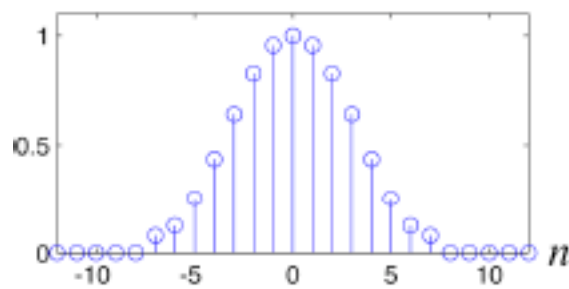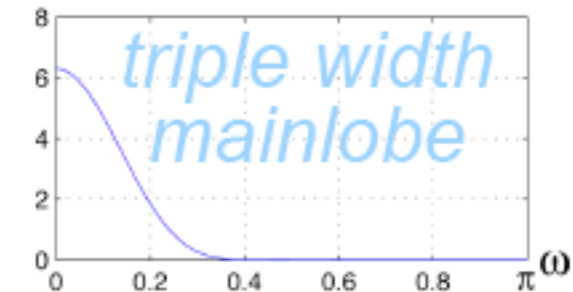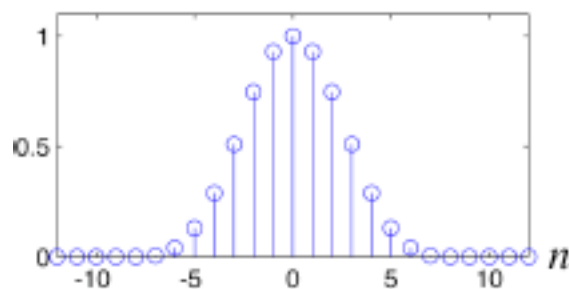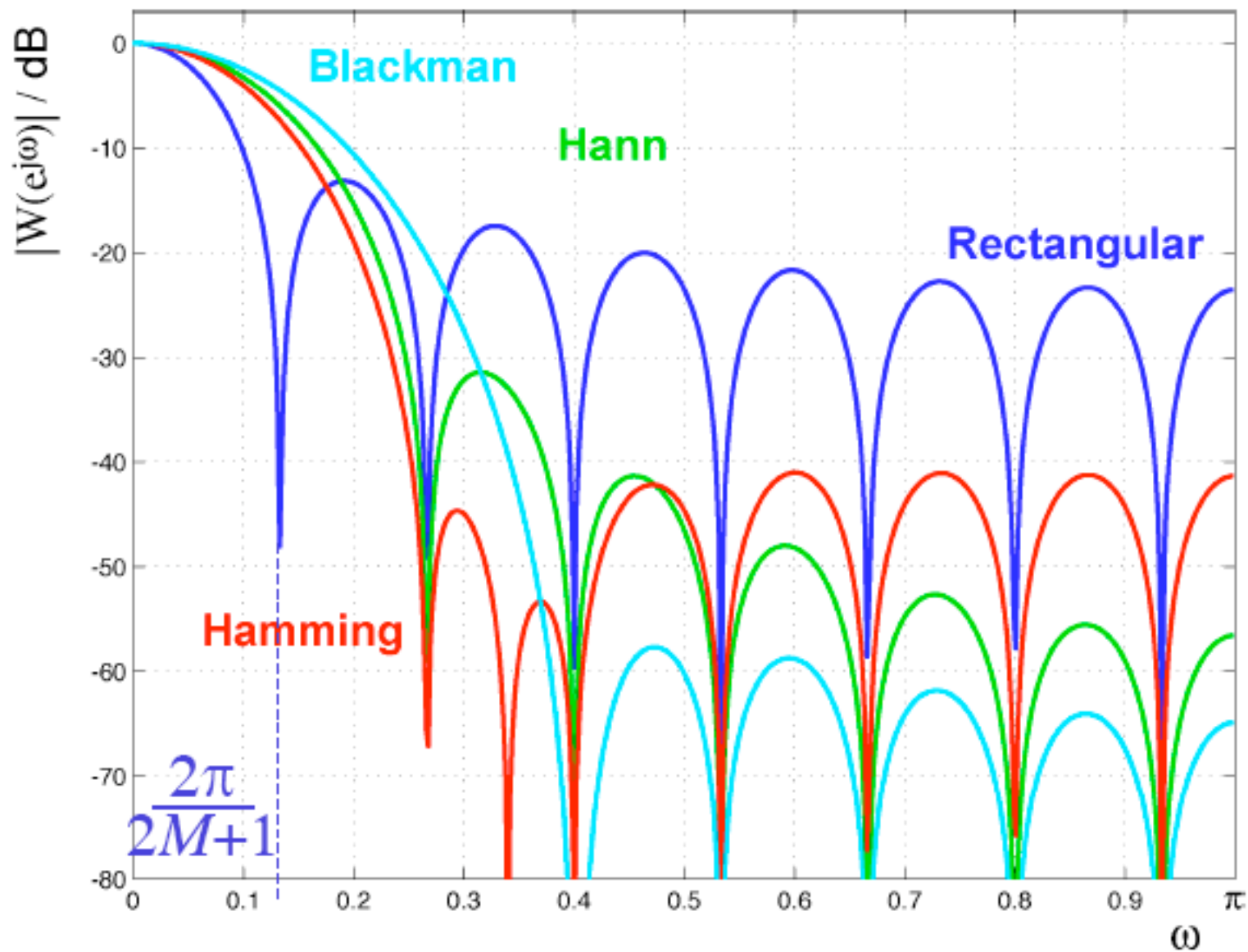
Windowed FIR Filter Example:

1. Get ideal filter impulse response:

$$\omega_c = 0.15\pi \quad \Rightarrow h_d[n] = \frac{\sin 0.15\pi n}{\pi n}$$

2. Get window function for truncation:

$$N = 25 \rightarrow M = 12 \quad (N=2M+1)$$

$$\Rightarrow w[n] = 0.54 + 0.46\cos\left(2\pi\frac{n}{25}\right) \quad -12 \le 12$$

3. Apply window:

$$h[n] = h_d[n]w[n]$$

$$= \frac{\sin 0.15\pi n}{\pi n}\left(0.54 + 0.46\cos\frac{2\pi n}{25}\right) \quad -12 \le 12$$
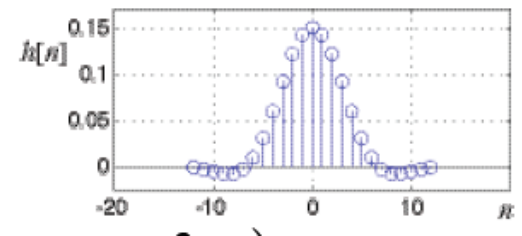
QDP 380 Stage 4
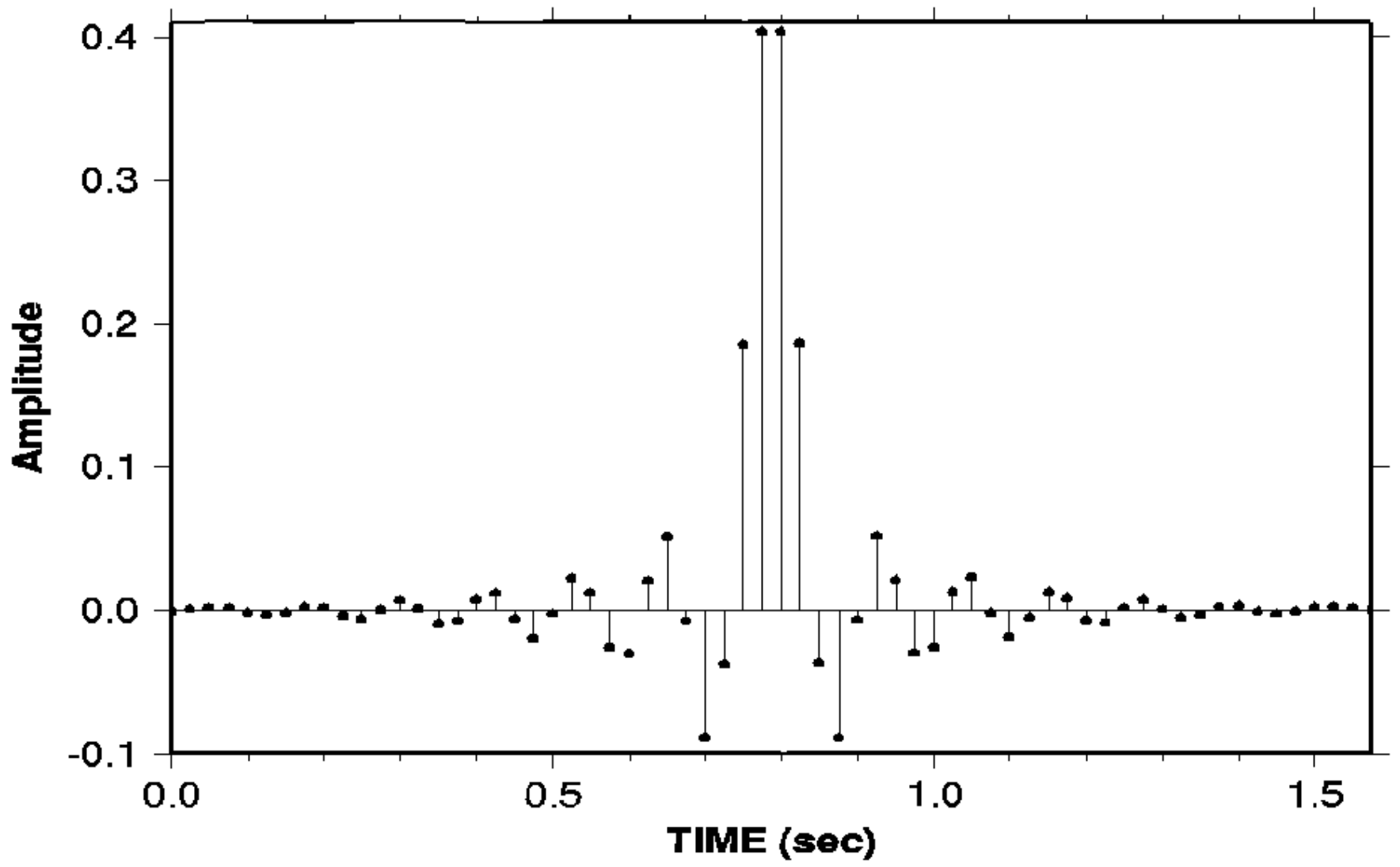
Zero Phase FIR Filter

Problem: Two-Sided IR

Cure: Change IR into Minimum Phase

Methods:

1) Add phase of Minimum Phase Filter to trace spectrum

2) Recursive Filtering of time inverted trace

0.013166

-0.015694

0.0   0.5   1.0   1.5   2.0   2.5   3.0   3.5

TIME [sec]

a)

b)

a)

b)

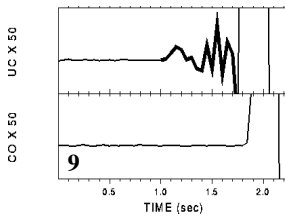The scatter plot shows Duration (sec) versus Seismic Moment (Nm) with a legend: FIR, Bez, Umd, EB, Iio, Ca.
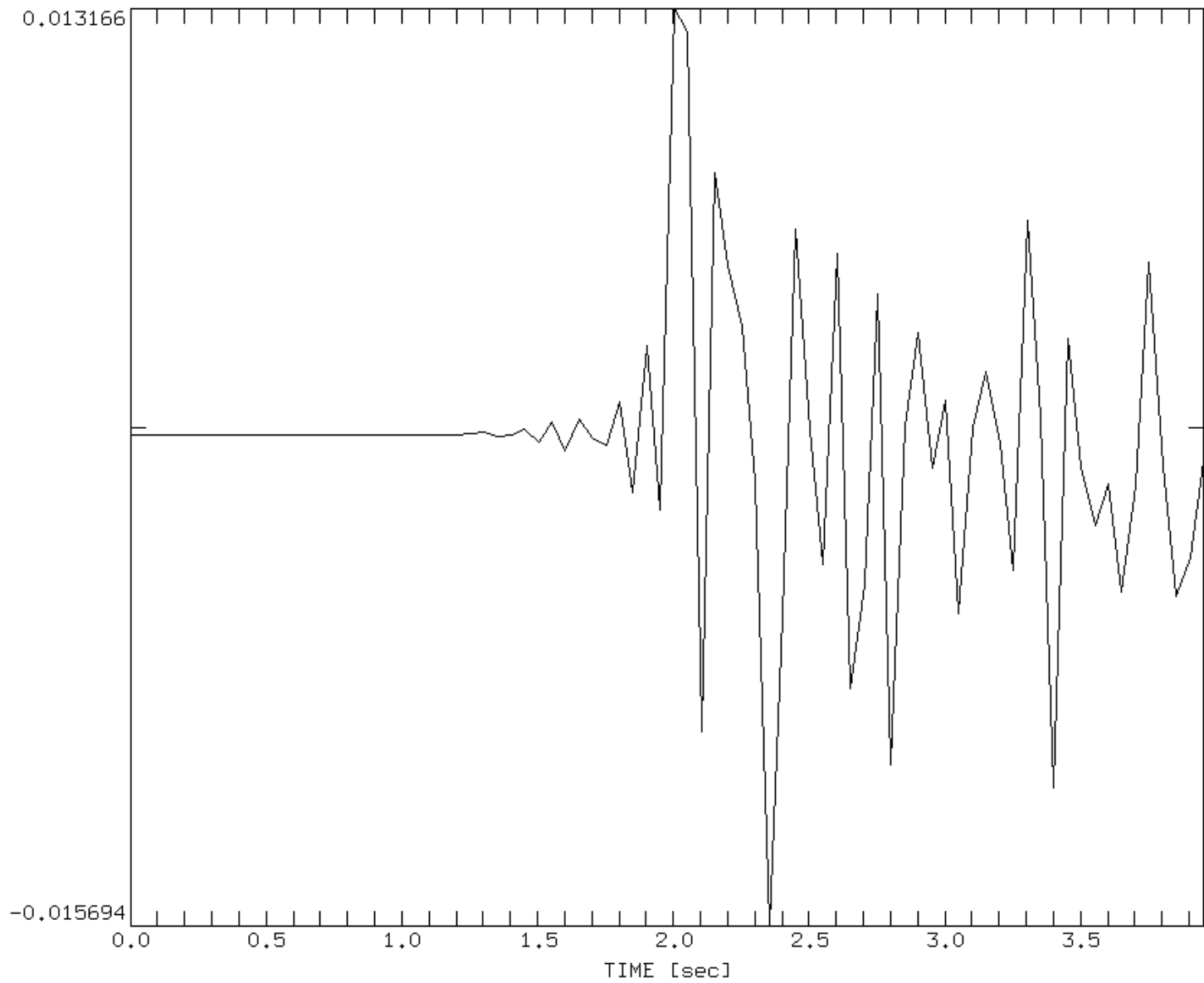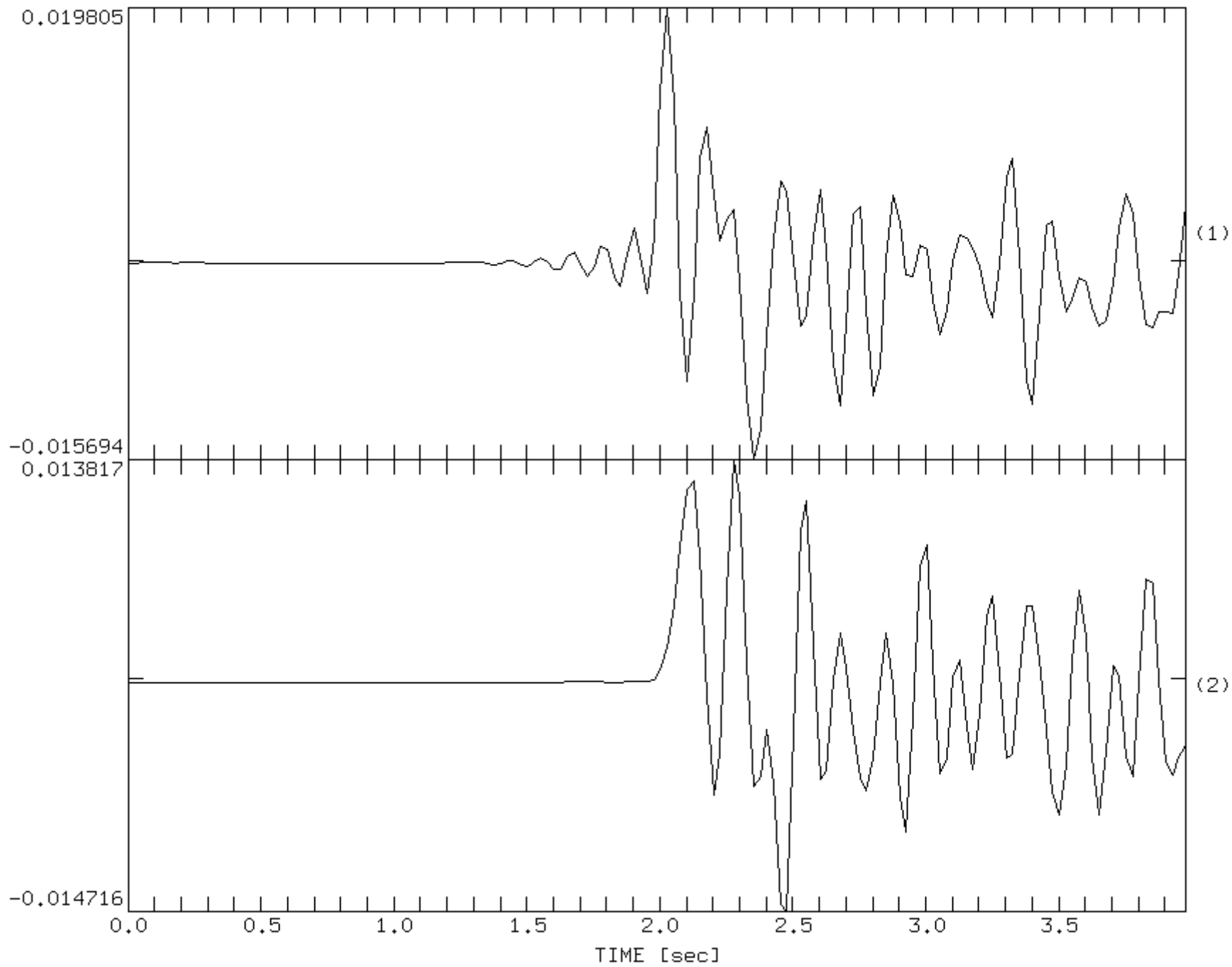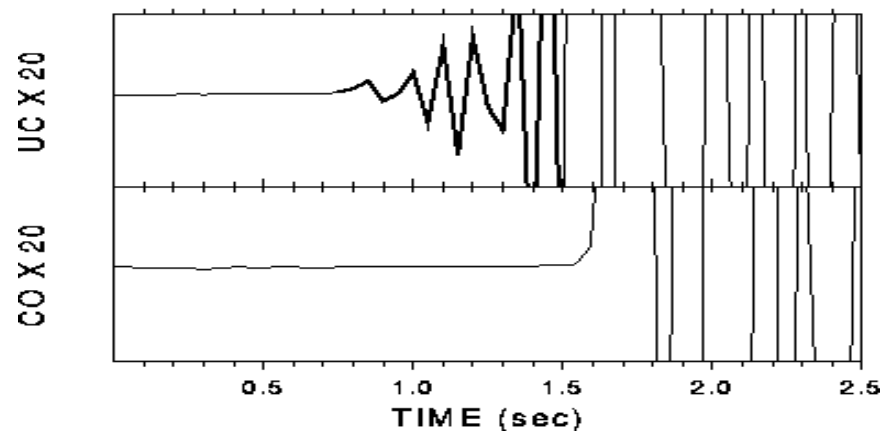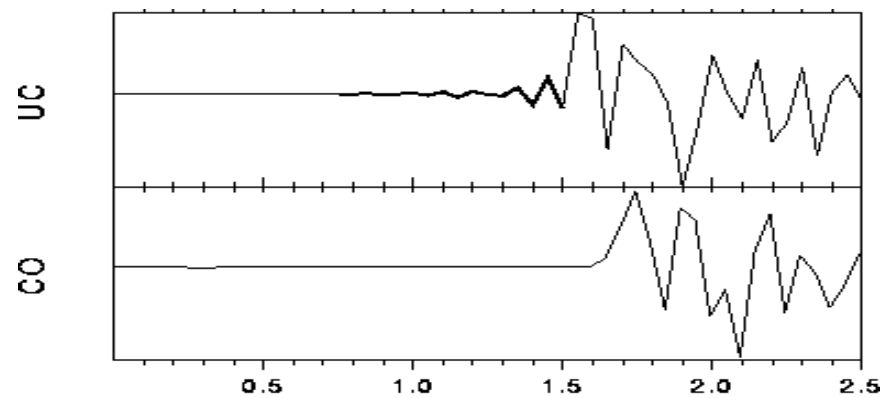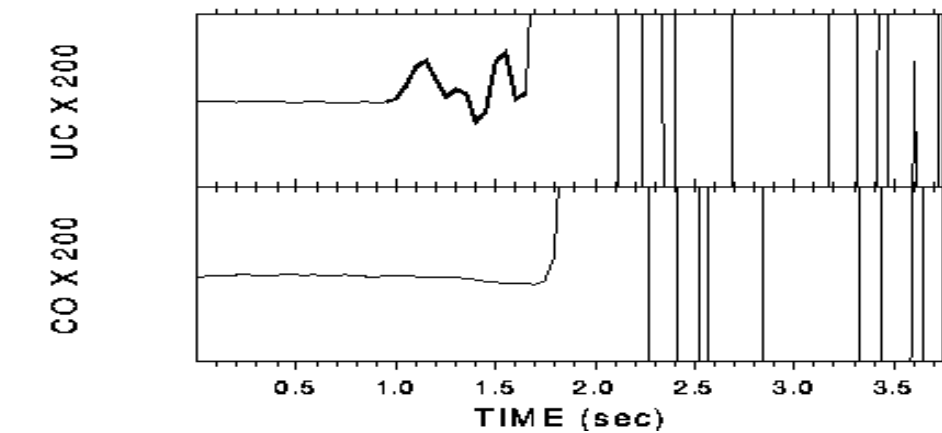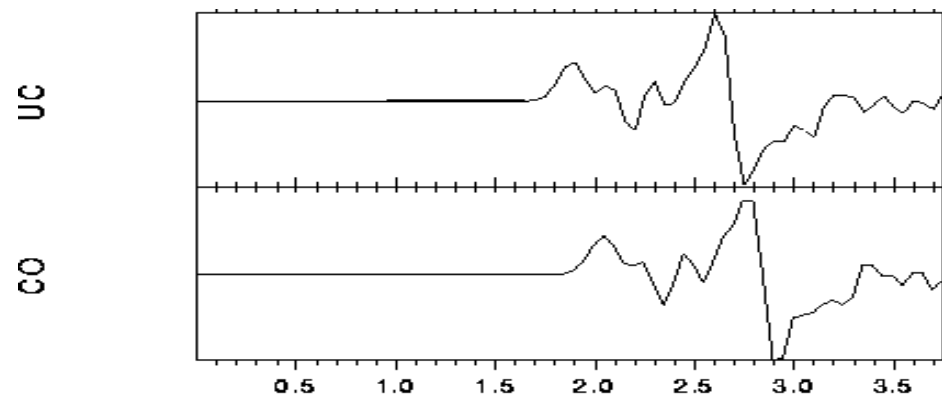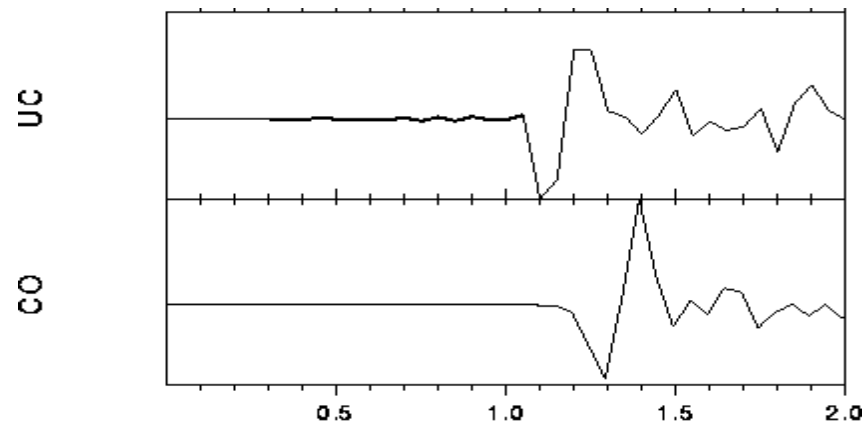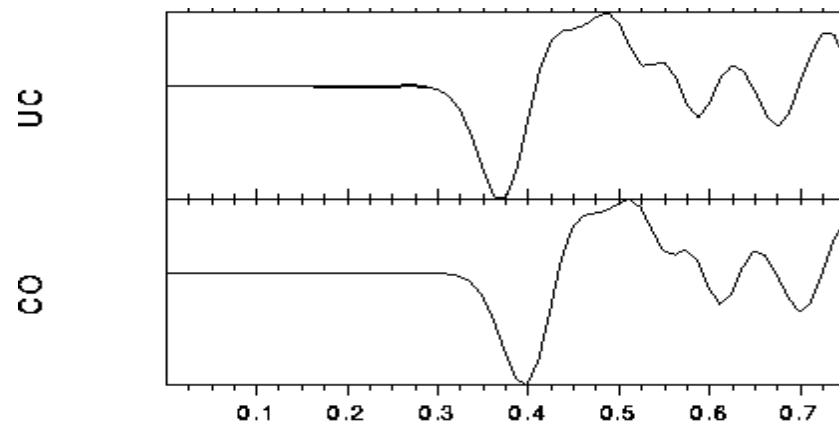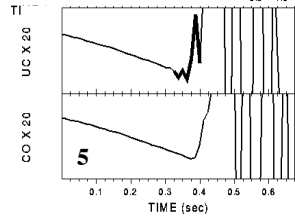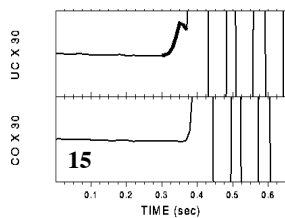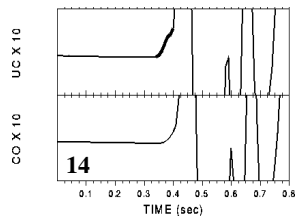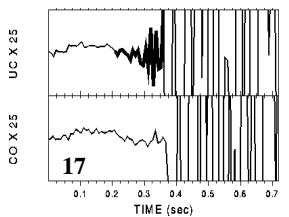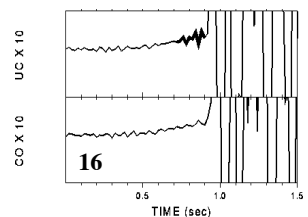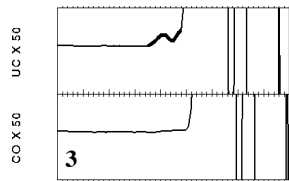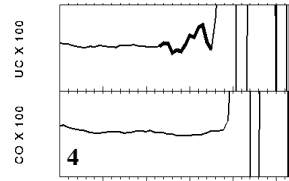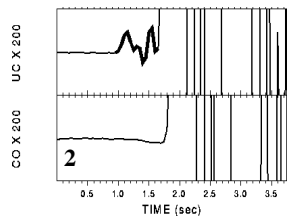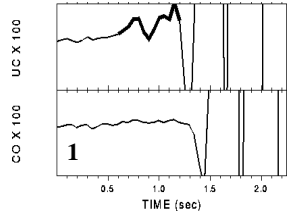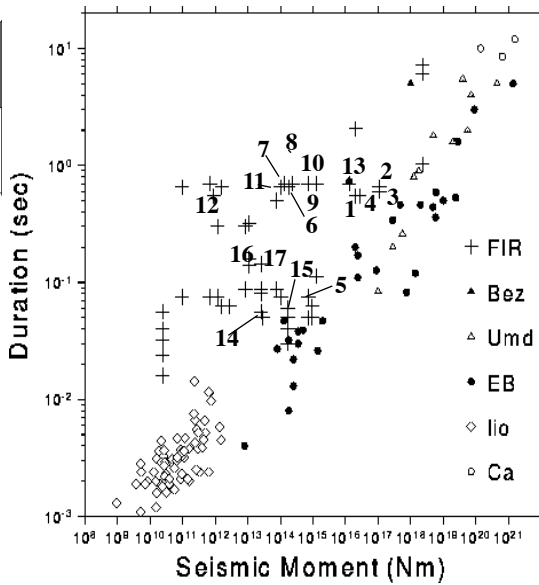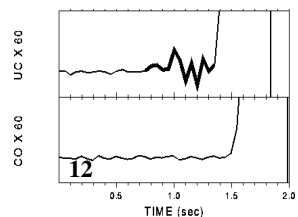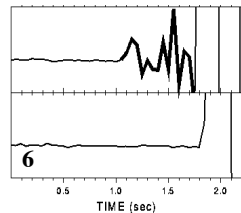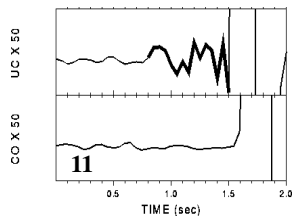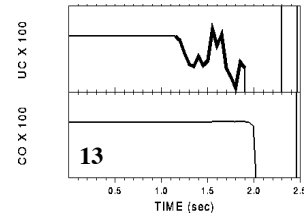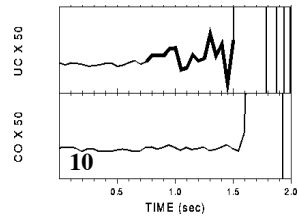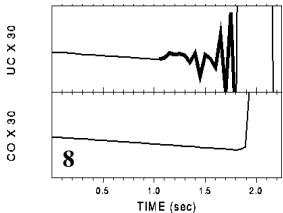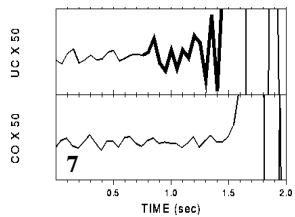
Conclusions

FIR filter generated precursory artefacts:

- **can become impossible to be identified visually**

- **can have similar scaling properties as nucleation phases**

    Zero - phase FIR filters in general

- **affect the determination of all onset properties (onset times, onset polarities)**

Consequence

For the interpretation of onset properties (onset times, onset polarities, nucleation phases, etc.) the acausal response of the zero-phase FIR filter has to be removed

but not

for waveform analysis.