Understanding the Earth by Stacking Receiver Functions:

The H-K Method

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ASI Kuwait, January 20, 2013

Who Am I?

Halfway along a 20 mile walk, Fimmvörðuháls vent, Iceland, April 2010

hiker, gardener, cook, softball outfielder, traveler

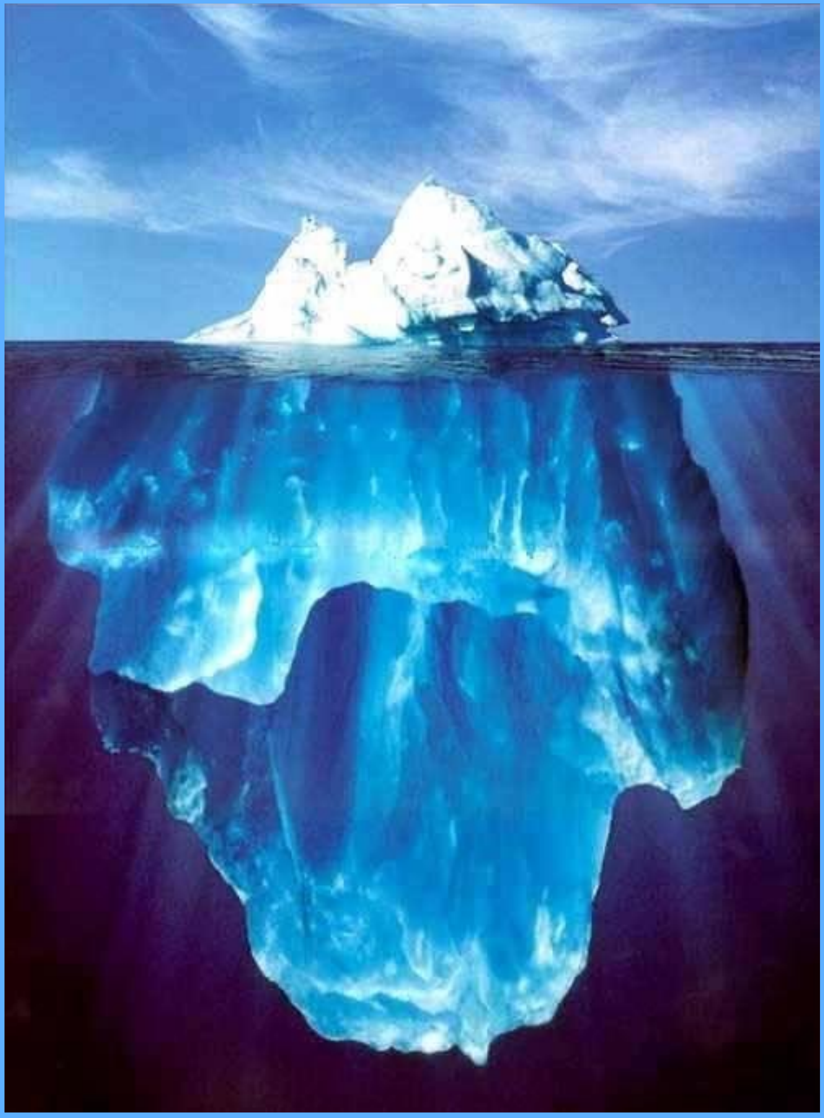
From: Florida, USA
Schools: U. South Carolina (B.S.)
U. Arizona (Ph.D.)

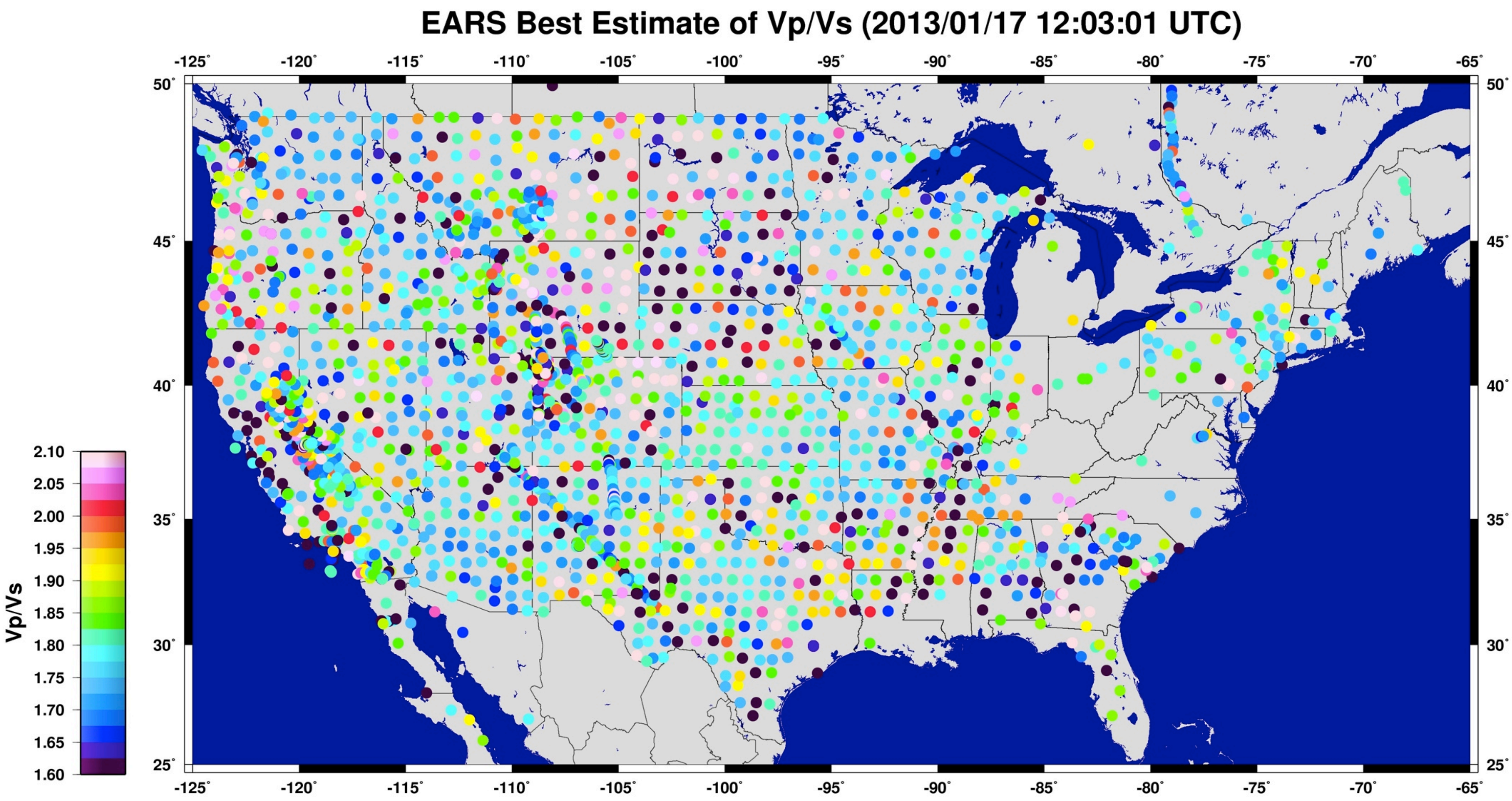
Currently (near) here:

Mt. Whitney, California,

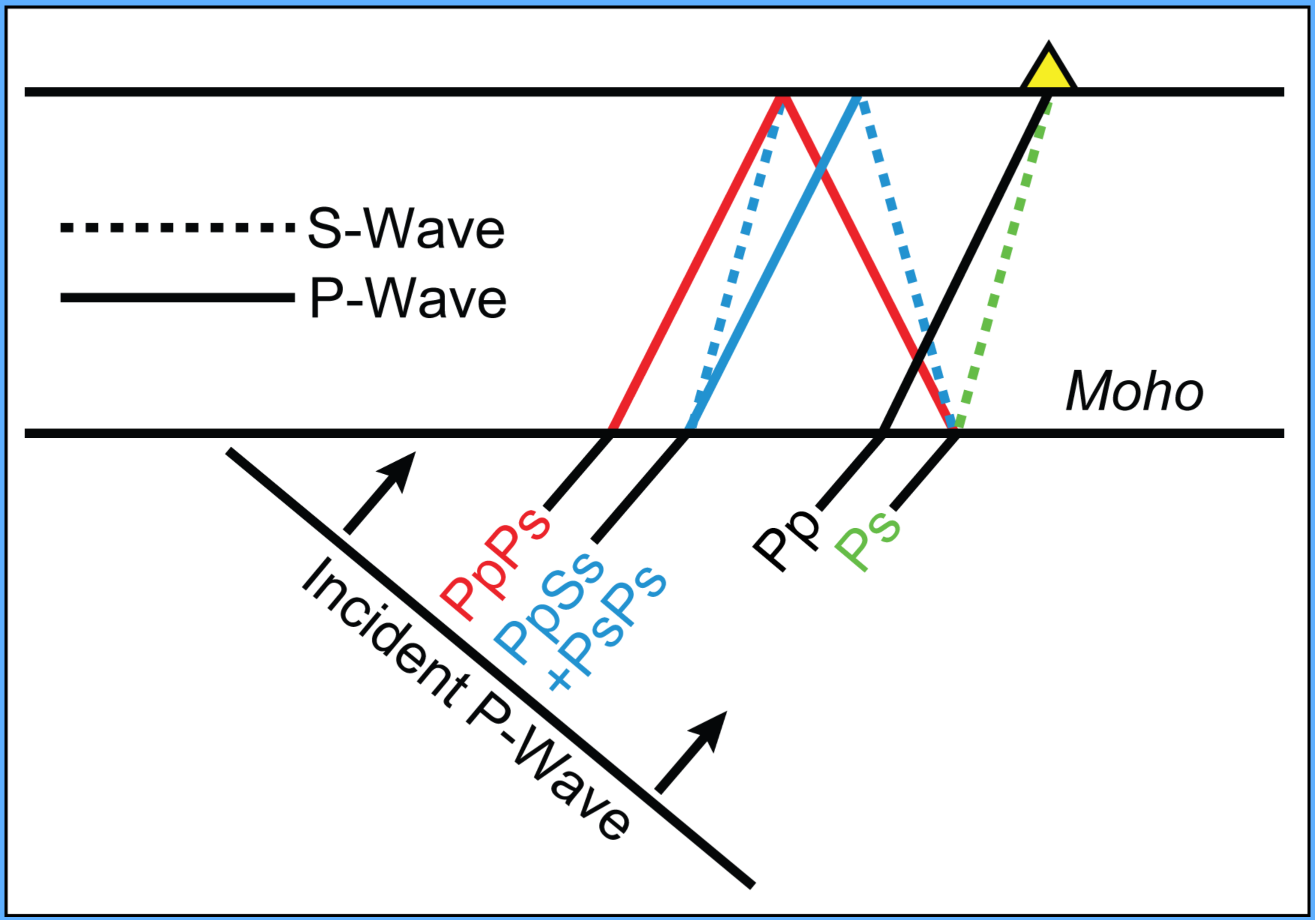
? ? ? ? ?

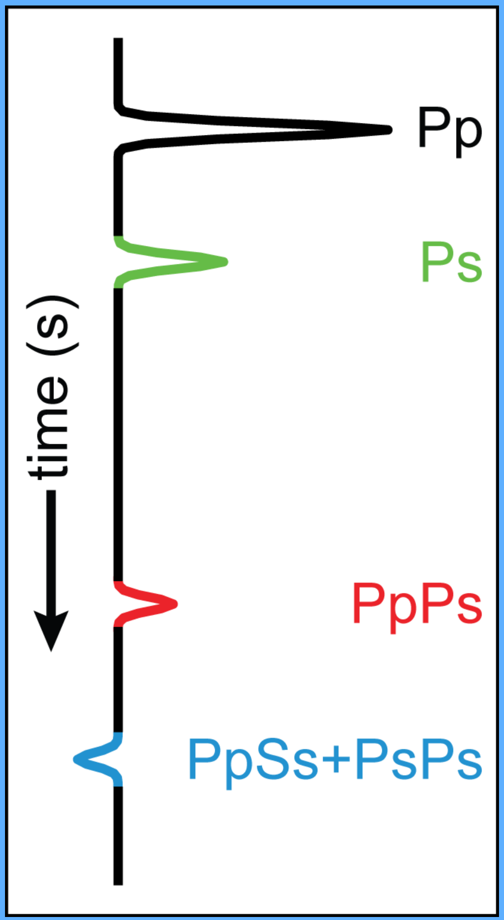
The tip of the iceberg…



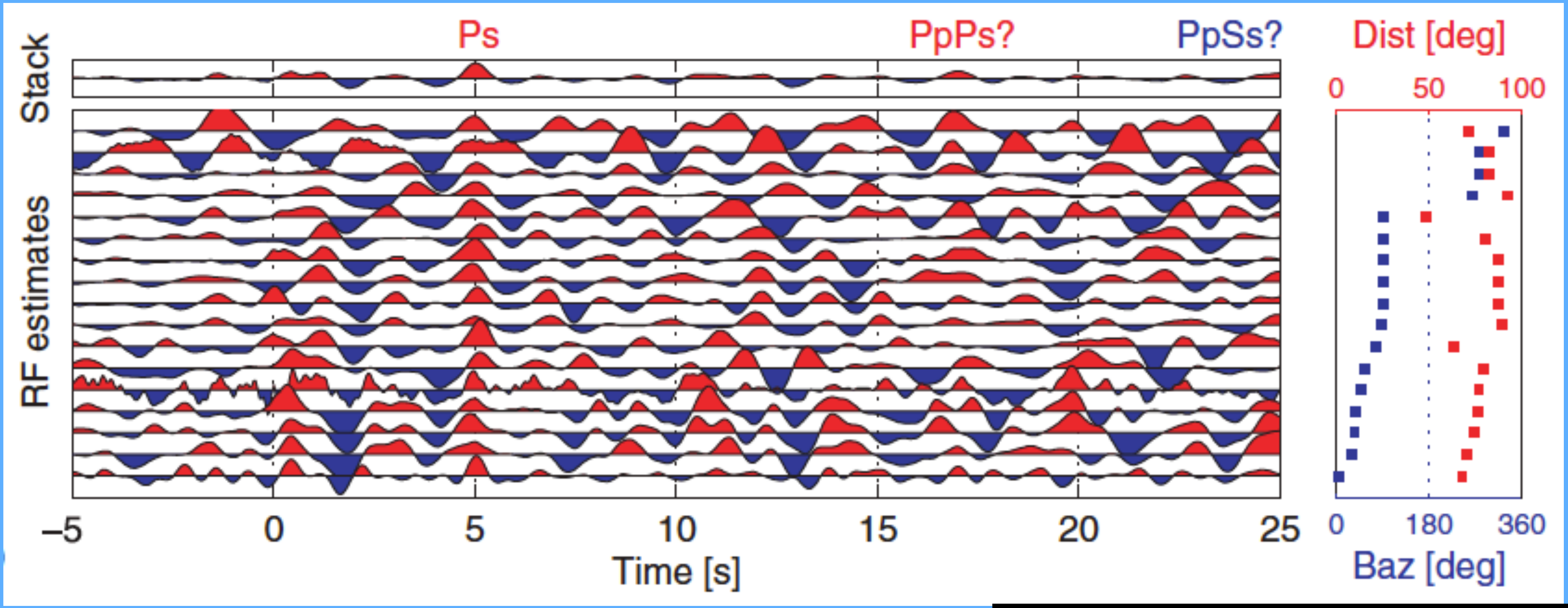

EARS Best Estimate of Vp/Vs (2013/01/17 12:03:01 UTC)
-125˚
-120˚
-115˚
-110˚
-105˚
-100˚
-95˚
-90˚
-85˚
-80˚
-75˚
-70˚
-65˚
50˚
45˚
40˚
35˚
30˚
25˚
Vp/Vs
2.10
2.05
2.00
1.95
1.90
1.85
1.80
1.75
1.70
1.65
1.60

Receiver Functions





Importance of Stacking

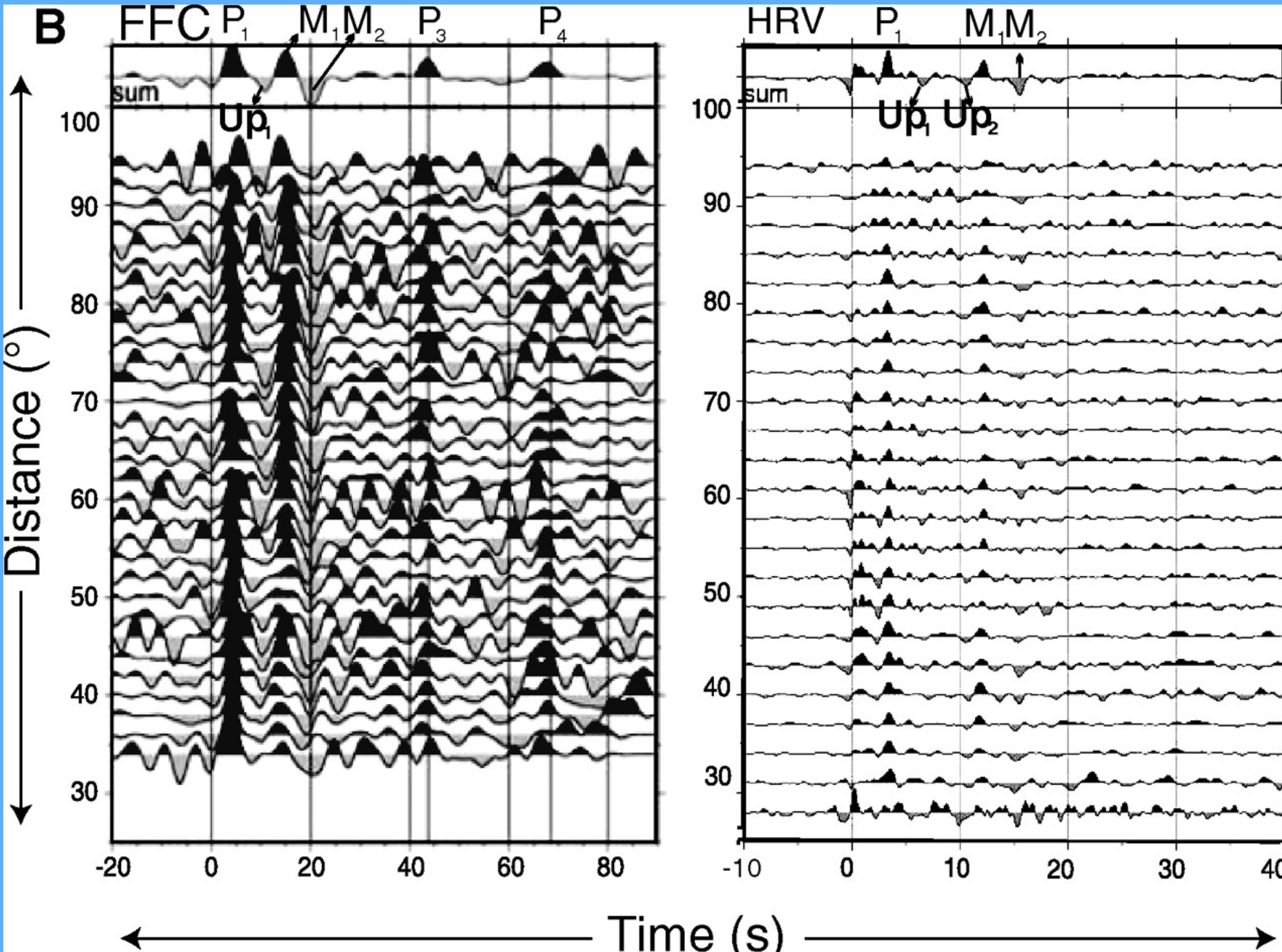


Svenningsen et al., 2007

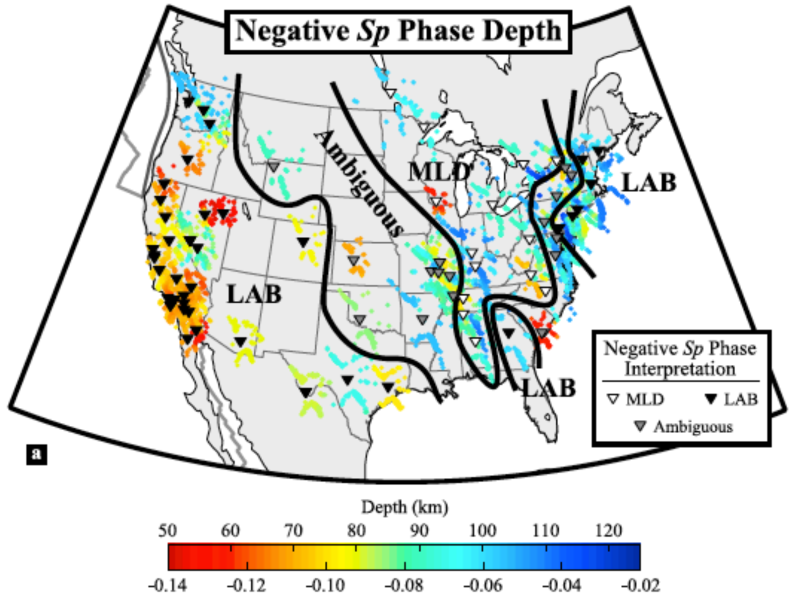
-Suppresses noise

-Emphasizes coherent signals

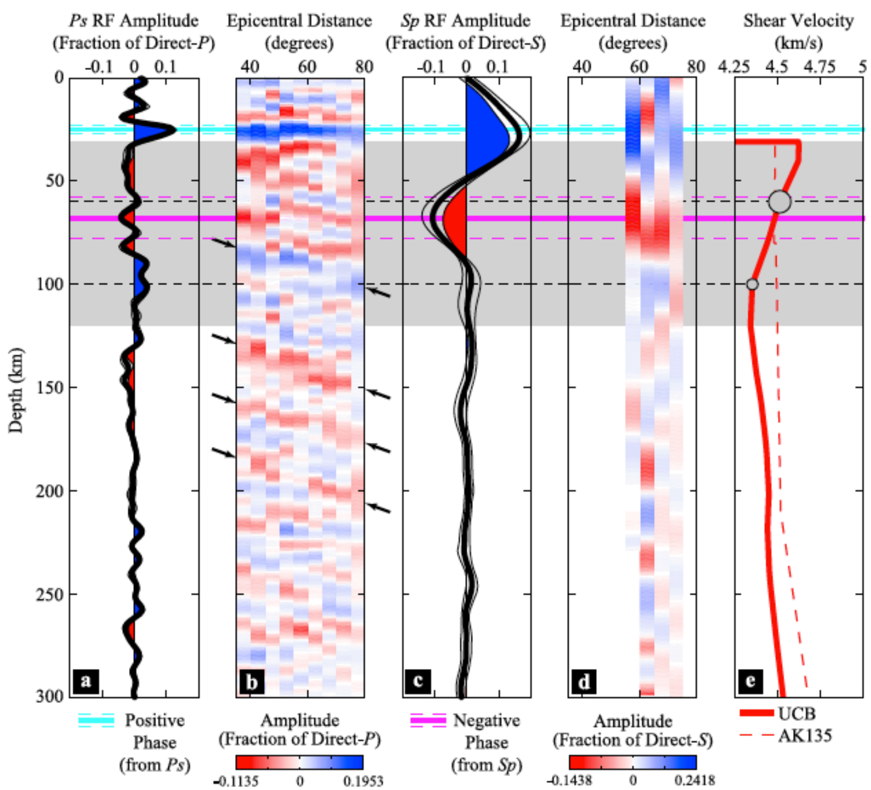
-S/N ratio grows by √# of RFs


B
FFC
P_1
M_1 M_2
P_3
P_4
sum
Up_1
100
90
80
70
60
50
40
30
Distance (°)
-20
0
20
40
60
80
HRV
P_1
M_1 M_2
sum
Up_1 Up_2
100
90
80
70
60
50
40
30
-10
0
10
20
30
40
Time (s)

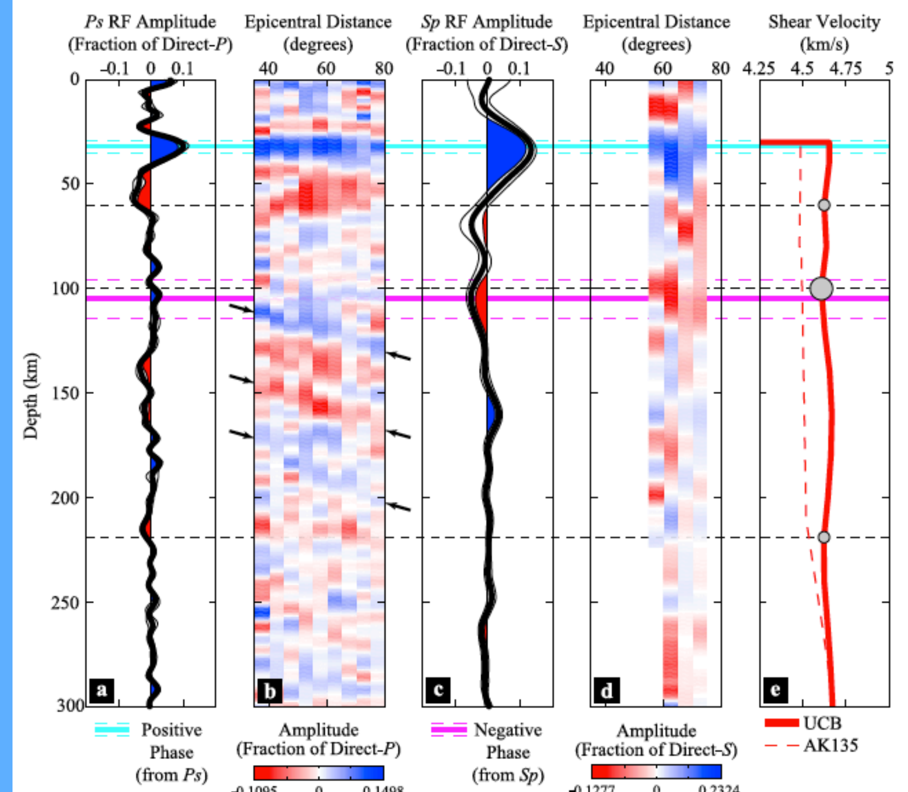
Abt et al., 2009



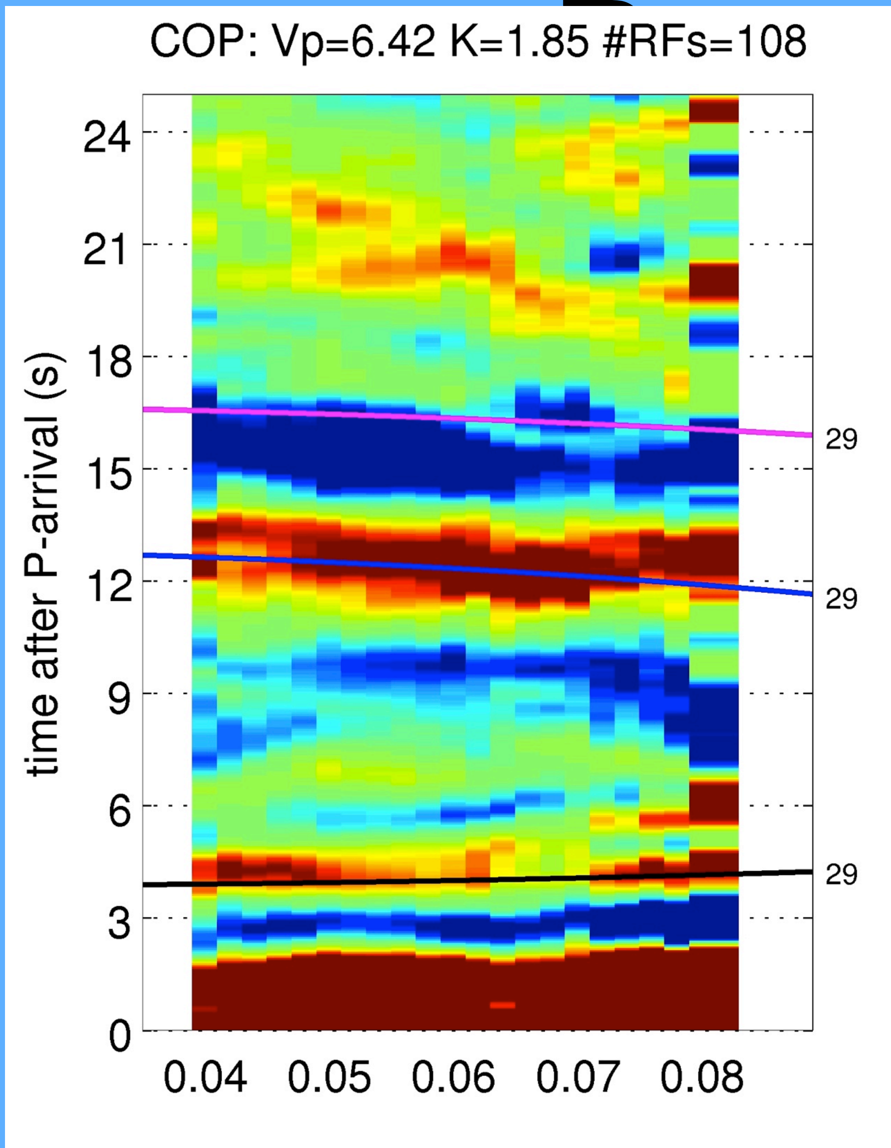
VTV (CI) - San Andreas Fault

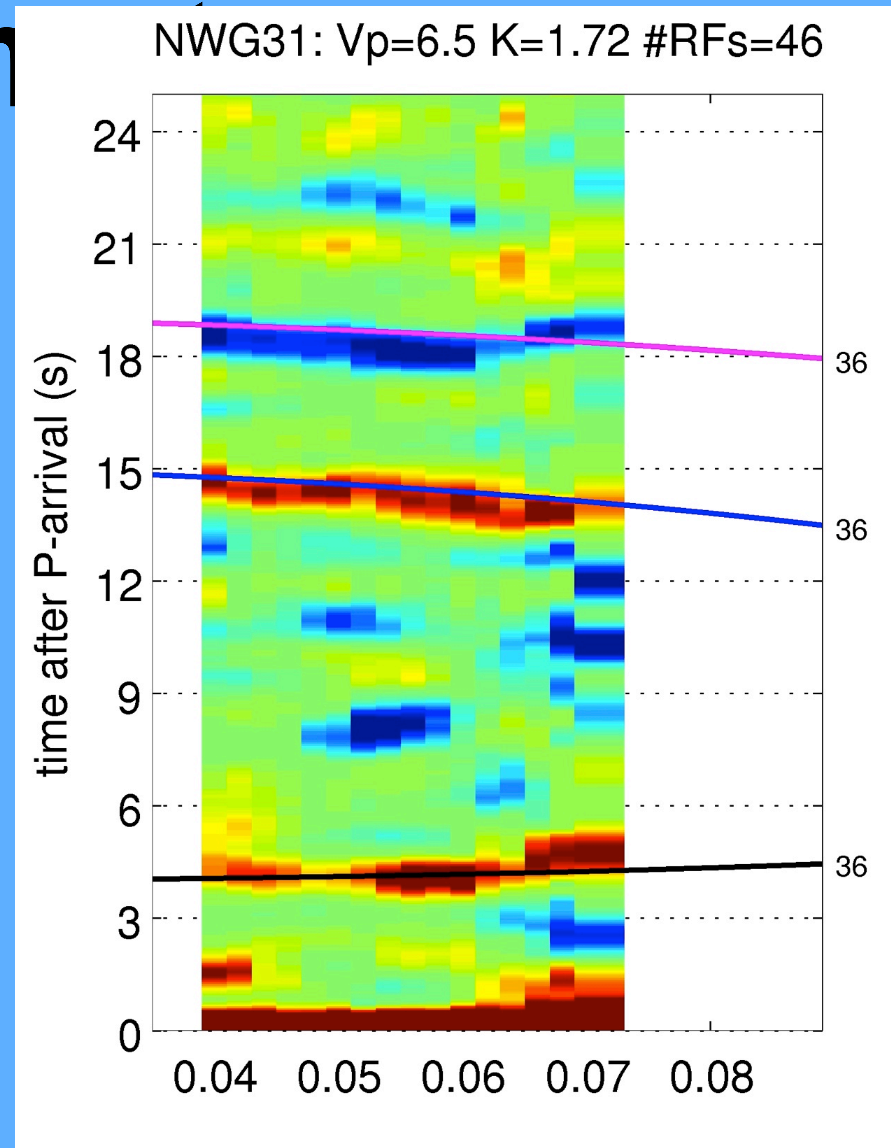


SDMD (LD) - Atlantic Margin, Maryland

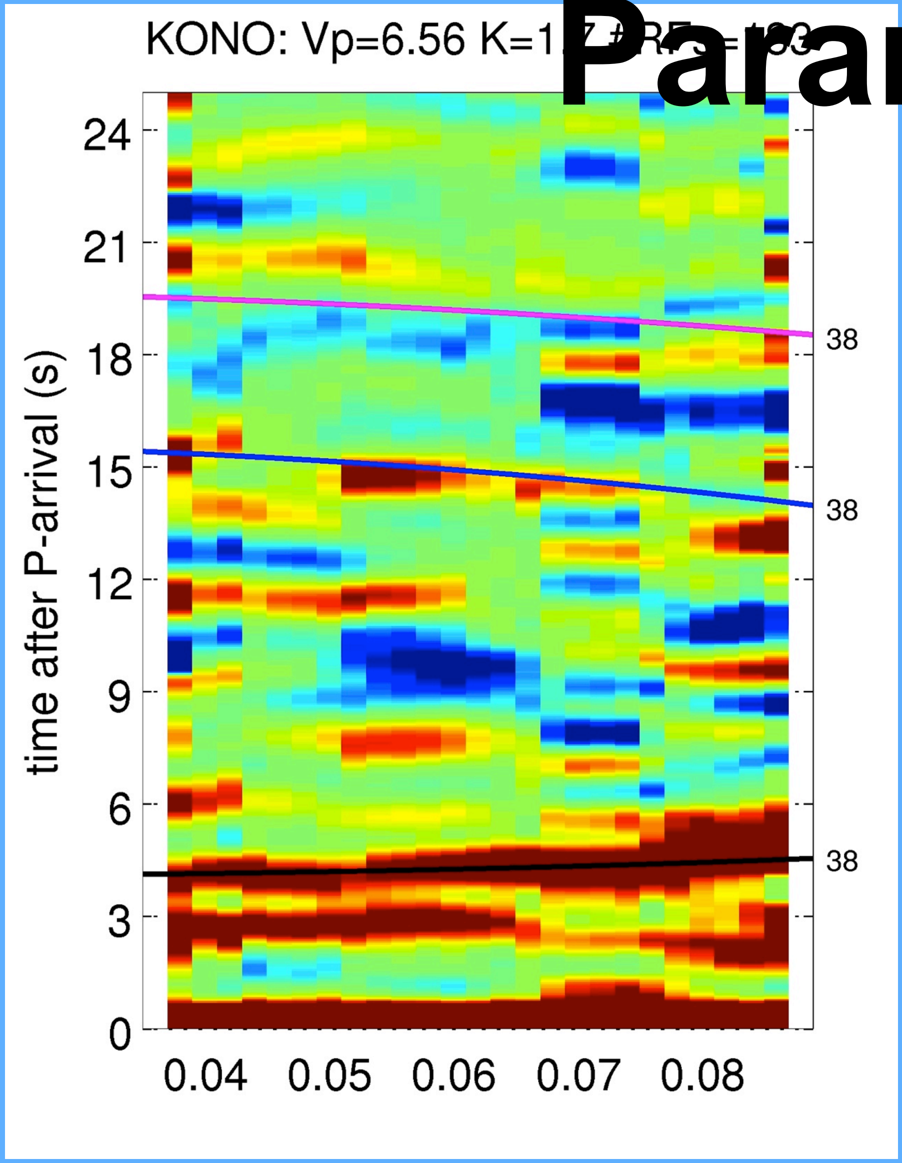


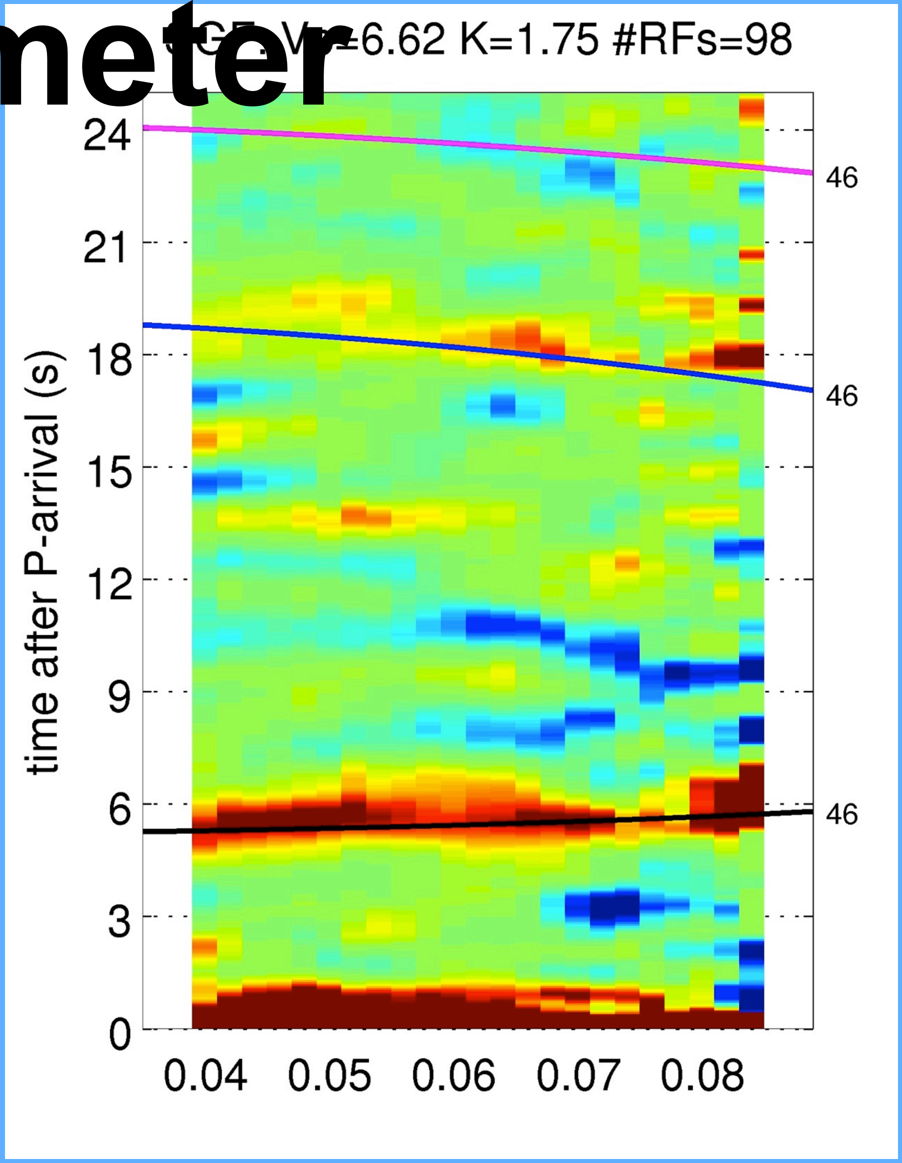
Arrivals vs. Ray



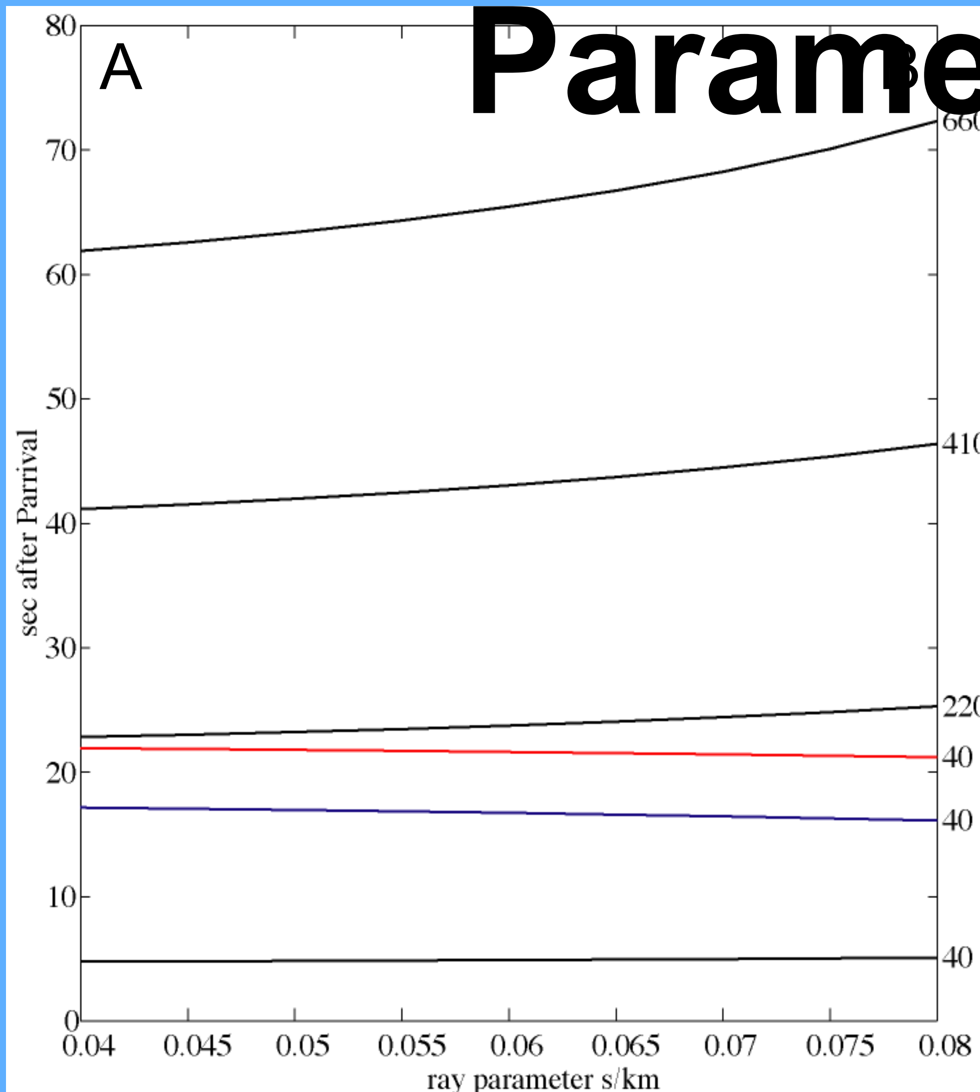


Arrivals vs. Ray Parameter

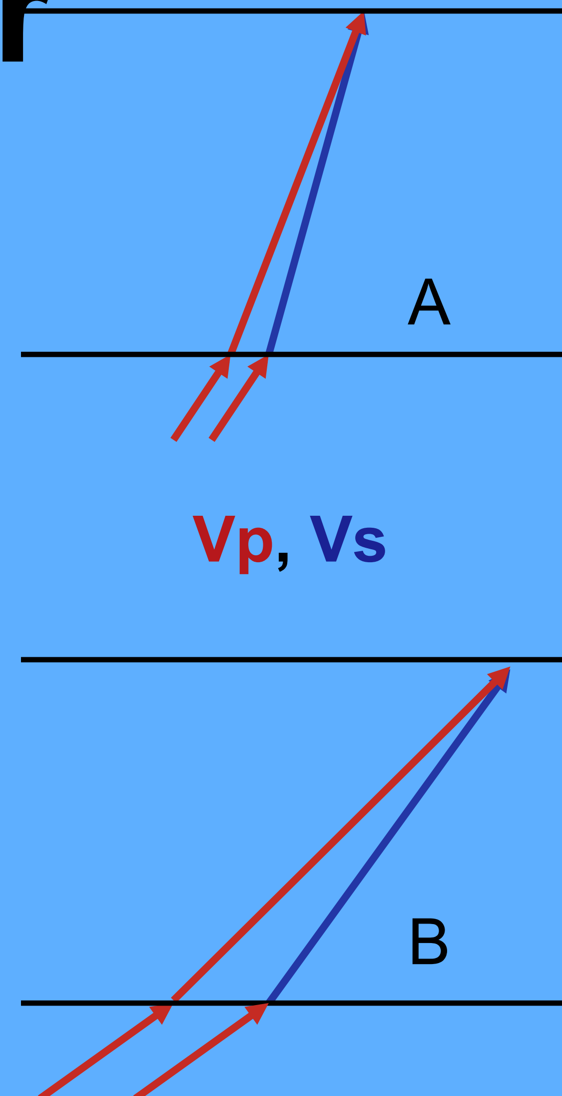




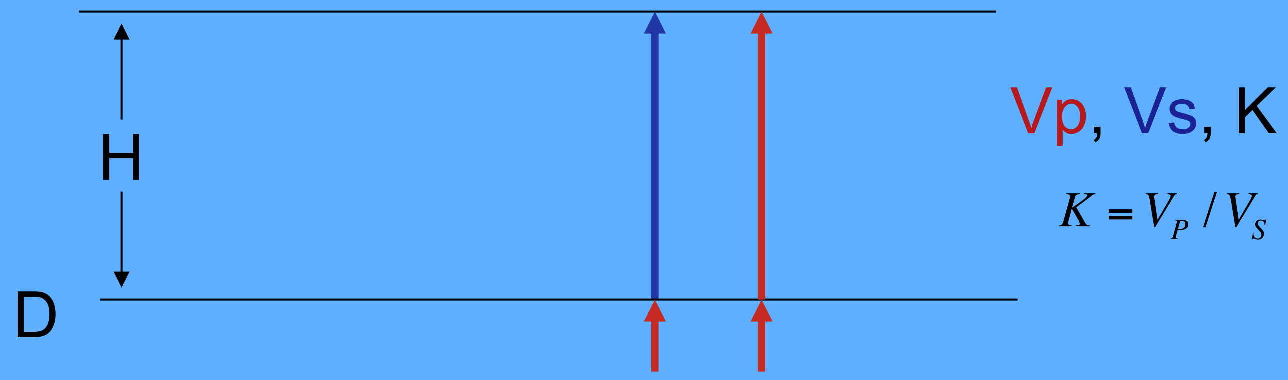
Arrivals vs. Ray Parameter



angle of incidence →



arrival time of vertically incident S-wave after P-wave

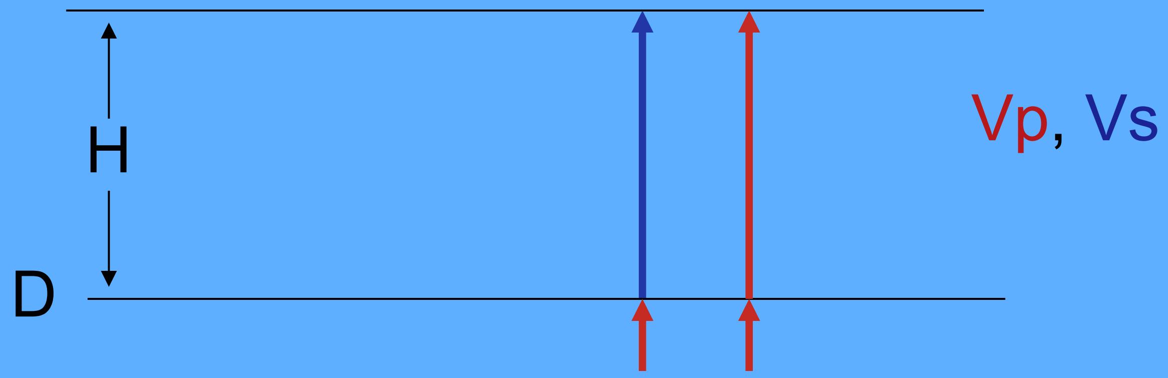


$$t_{Ps} = \int_D^0 \frac{1}{V_S(z)} - \frac{1}{V_P(z)} dz$$

Easy! Take the integral of the S-wave slowness minus P-wave slowness

Within a layer of constant velocities

$$t_{Ps} = \Delta T_{Ps} = \frac{\Delta z}{V_S} - \frac{\Delta z}{V_P}$$

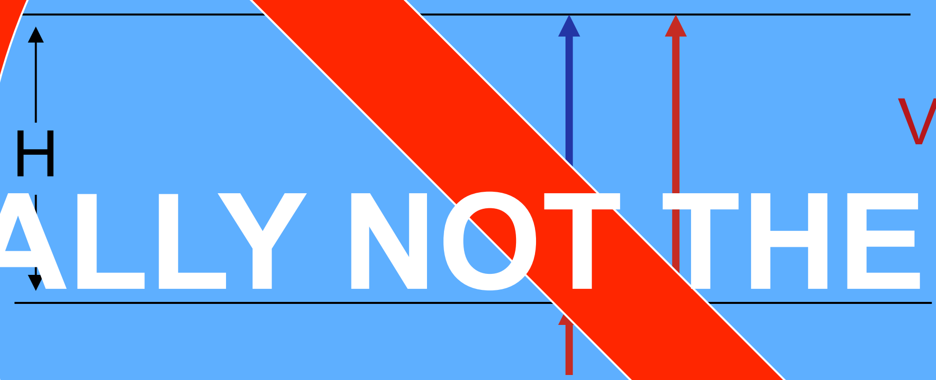


Find the expected arrival time for vertically traveling P and S waves in layer with Vs=3.5 km/s, Vp=6.4 km/s, H=40 km.

$$t_{Ps} = \frac{40km}{3.5km/s} - \frac{40km}{6.4km/s} = 11.4s - 6.25s = 5.1s$$

Within a la… velocities

$$= \Delta T_{Ps} = \frac{\Delta z}{V_S} - \frac{\Delta z}{V_P}$$

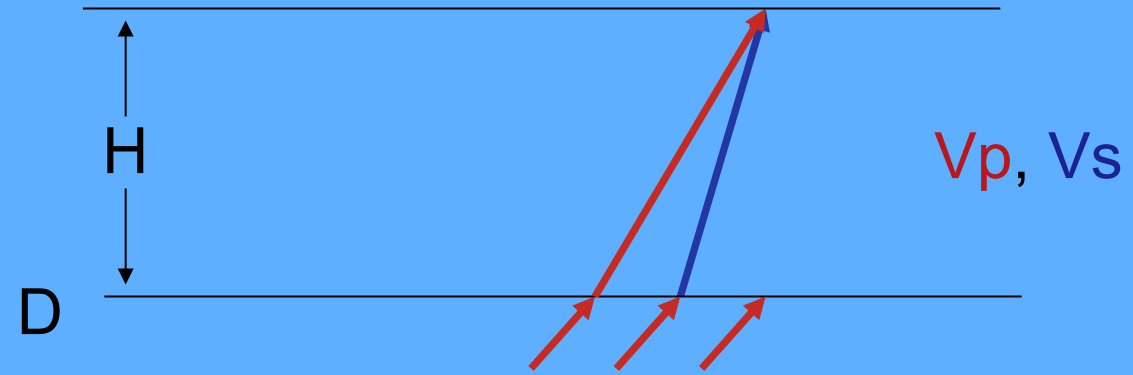


USUALLY NOT THE CASE!

Find the …ected arrival time for ver… traveling … and S waves in …er with Vs=3.5 km/s, Vp=6… /s, …0 km.

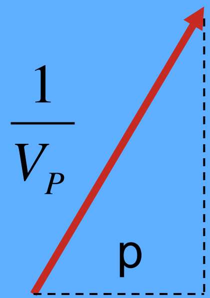
$$t_{Ps} = \frac{40k\ldots}{3.5km/s} \ldots \frac{40km}{\ldots} = 11.4 \ldots .25s = 5.1s$$

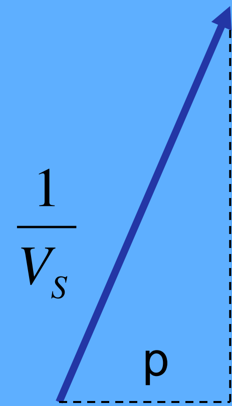
Arrival time of a non-vertically incident Pds wave



$$t_{Ps} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} - \sqrt{V_P(z)^{-2} - p^2}\, dz$$

Need to know Vp, Vs, and p (slowness) of the incident P-wave





Vp, Vs, K

vertical component of P slowness $= \sqrt{V_P^{-2} - p^2}$

vertical component of S slowness $= \sqrt{V_S^{-2} - p^2}$

p is constant

$$t_{Ps} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} - \sqrt{V_P(z)^{-2} - p^2}\, dz$$

$$= 40 * \sqrt{3.5^{-2} - 0.06^2} - \sqrt{6.4^{-2} - 0.06^2}$$

$$= 5.4\,\mathrm{s}$$

we have done examples solving the forward problem of calculating what TIME a phase from a given depth would arrive

Given depth, Vp, Vs, and p, we found t.

$$t_{Ps} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} - \sqrt{V_P(z)^{-2} - p^2}\, dz$$

now we want to solve for what depth a phase was produced depending on its arrival time

with constant Vs and Vp that do not depend on depth arrival time equation becomes:

$$t_{Ps} = z\left(\sqrt{V_S^{-2} - p^2} - \sqrt{V_P^{-2} - p^2}\right)$$

solving then for depth z

$$\frac{t_{Ps}}{\sqrt{V_S^{-2} - p^2} - \sqrt{V_P^{-2} - p^2}} = z$$

we pick the arrival time of the Ps phase, calculate p, and assume Vp, and Vs

$$\frac{t_{Ps}}{\sqrt{V_S^{-2} - p^2} - \sqrt{V_P^{-2} - p^2}} = z$$

Pds arrival time = 5 s, p = 0.06 s/km and Vp=6.4 km/s and Vs =3.5 km/s

$$\frac{5}{\sqrt{3.5^{-2} - 0.06^2} - \sqrt{6.4^{-2} - 0.06^2}} = z$$

$$\frac{5}{\sqrt{3.5^{-2} - 0.06^2} - \sqrt{6.4^{-2} - 0.06^2}} = z$$

z = 37 km

how sensitive is this value of z to our assumptions?

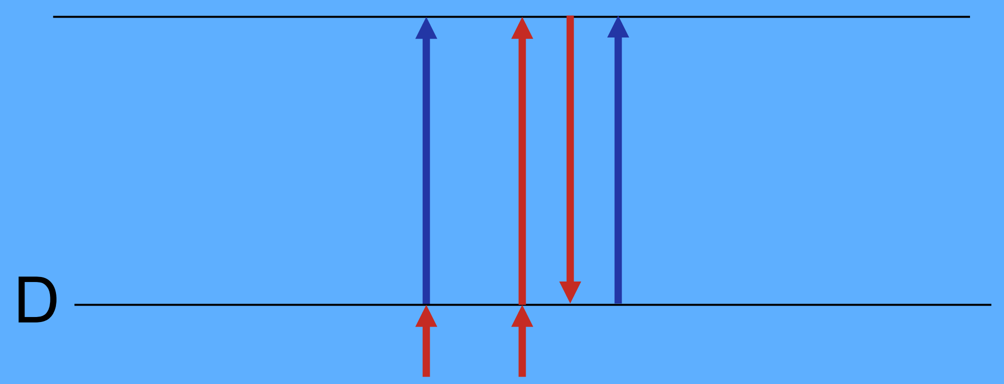
Vary Vp to observe the SMALL effect on crustal thickness estimates

Vp (km/s)	K (Vp/Vs)	Vs (km)	p (s/km)	H (km)
6.10	1.73	3.53	0.06	40.12
6.14	1.73	3.55	0.06	40.36
6.18	1.73	3.57	0.06	40.60
6.22	1.73	3.60	0.06	40.84
6.26	1.73	3.62	0.06	41.08
6.30	1.73	3.64	0.06	41.32
6.34	1.73	3.66	0.06	41.56
6.38	1.73	3.69	0.06	41.79
6.42	1.73	3.71	0.06	42.03
6.46	1.73	3.73	0.06	42.27
6.50	1.73	3.76	0.06	42.50
6.54	1.73	3.78	0.06	42.74
6.58	1.73	3.80	0.06	42.97
6.62	1.73	3.83	0.06	43.21
6.66	1.73	3.85	0.06	43.44

Vary Vp/Vs to observe the LARGE effect on crustal thickness estimates

Vp (km/s)	K (Vp/Vs)	Vs (km)	p (s/km)	H (km)
6.40	1.70	3.76	0.06	43.67
6.40	1.72	3.72	0.06	42.48
6.40	1.74	3.68	0.06	41.36
6.40	1.76	3.64	0.06	40.29
6.40	1.78	3.60	0.06	39.28
6.40	1.80	3.56	0.06	38.31
6.40	1.82	3.52	0.06	37.40
6.40	1.84	3.48	0.06	36.52
6.40	1.86	3.44	0.06	35.69
6.40	1.88	3.40	0.06	34.90
6.40	1.90	3.37	0.06	34.13
6.40	1.92	3.33	0.06	33.41
6.40	1.94	3.30	0.06	32.71
6.40	1.96	3.27	0.06	32.04
6.40	1.98	3.23	0.06	31.40

arrival time of vertically incident PpPs-wave after P-wave



Vp, Vs, K

$$K = V_P / V_S$$

$$t_{Ps} = \int_D^0 \frac{1}{V_P(z)} + \frac{1}{V_P(z)} + \frac{1}{V_S(z)} - \frac{1}{V_P(z)} dz$$

$$t_{Ps} = \int_D^0 \frac{1}{V_P(z)} + \frac{1}{V_S(z)} dz$$

as we saw for the direct Ps wave the arrival time is:

$$t_{Ps} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} - \sqrt{V_P(z)^{-2} - p^2}\,dz$$

following the progression explained for the PpPs reverberation phase, its arrival time is:

$$t_{PpPs} = \int_D^0 \sqrt{V_S(z)^{-2} - p^2} + \sqrt{V_P(z)^{-2} - p^2}\,dz$$

similarly the PsPs + PpSs arrival time is:

$$t_{PsPs+PpSs} = \int_D^0 2\sqrt{V_S(z)^{-2} - p^2}\,dz$$

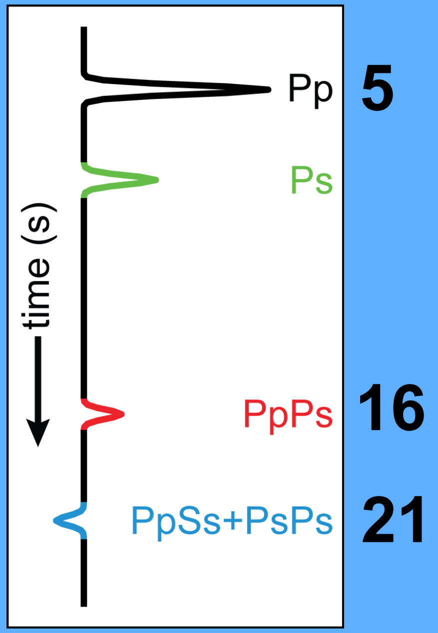
we pick the arrival times of the Pds, PpPs, and PsPs+PpSs phases, calculate p, and assume Vp, and Vs

$$\frac{16}{\sqrt{3.5^{-2}-0.06^2}+\sqrt{6.4^{-2}-0.06^2}} = z_{PpPs}$$

$$37.7km = z_{PpPs}$$

$$\frac{21}{2\sqrt{3.5^{-2}-0.06^2}} = z_{PsPs+PpSs}$$

$$37.6km = z_{PsPs+PpSs}$$



Pds time = 5 s, PpPs = 16 s, PsPs+PpSs time = 21 s, p= 0.06 s/km and Vp=6.4 km/s and Vs =3.5 km/s

$$37.6km = z_{PsPs+PpSs}$$

$$37.7km = z_{PpPs}$$

$$37.0km = z_{Ps}$$

How do we account for these differences?

Are they all resulting from an interface at the same depth?

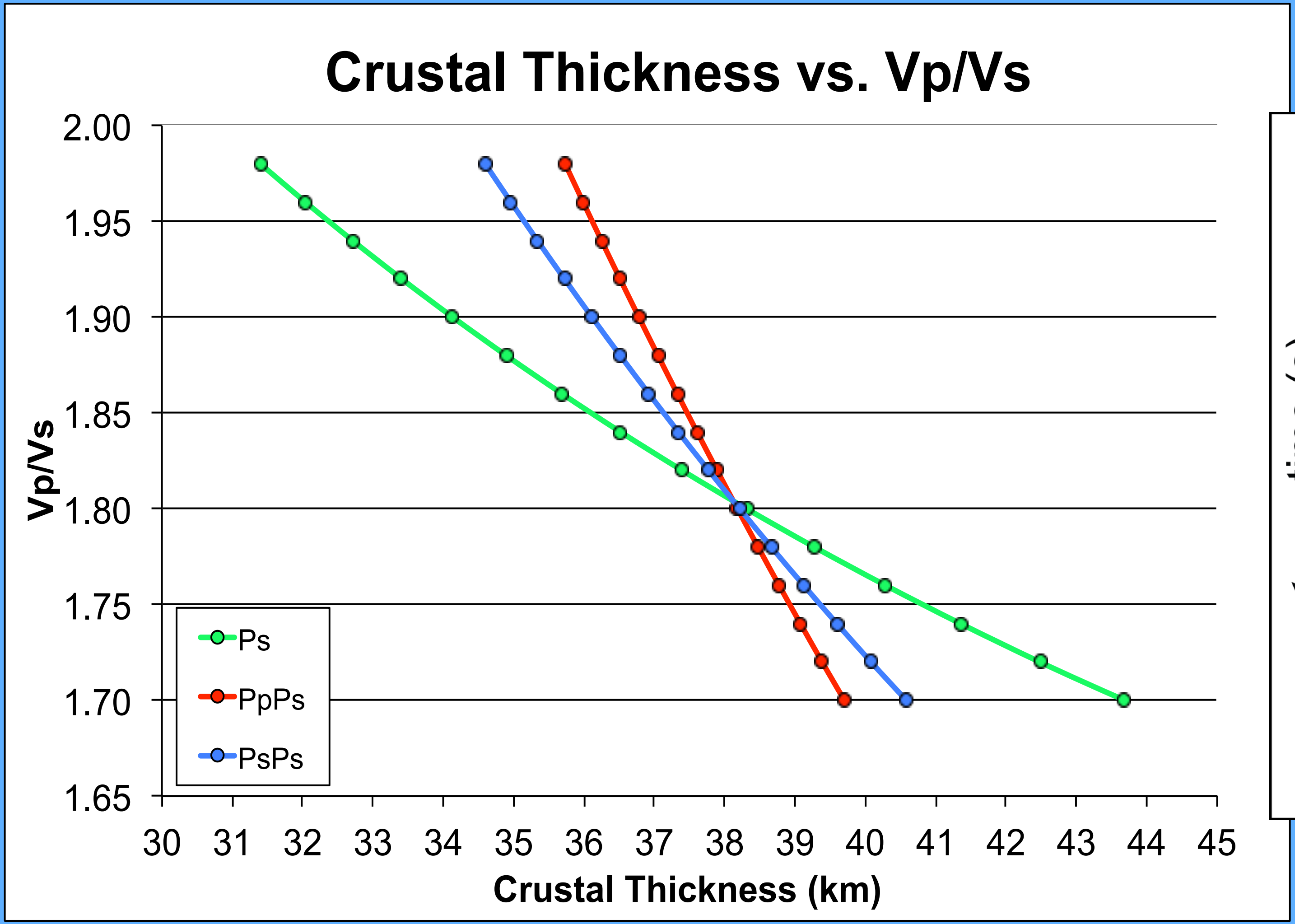
Were the phases all picked correctly?

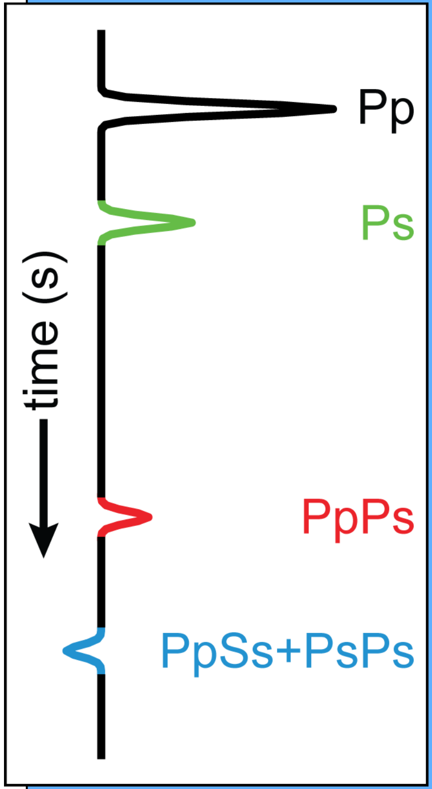
Could we have made an incorrect assumption?

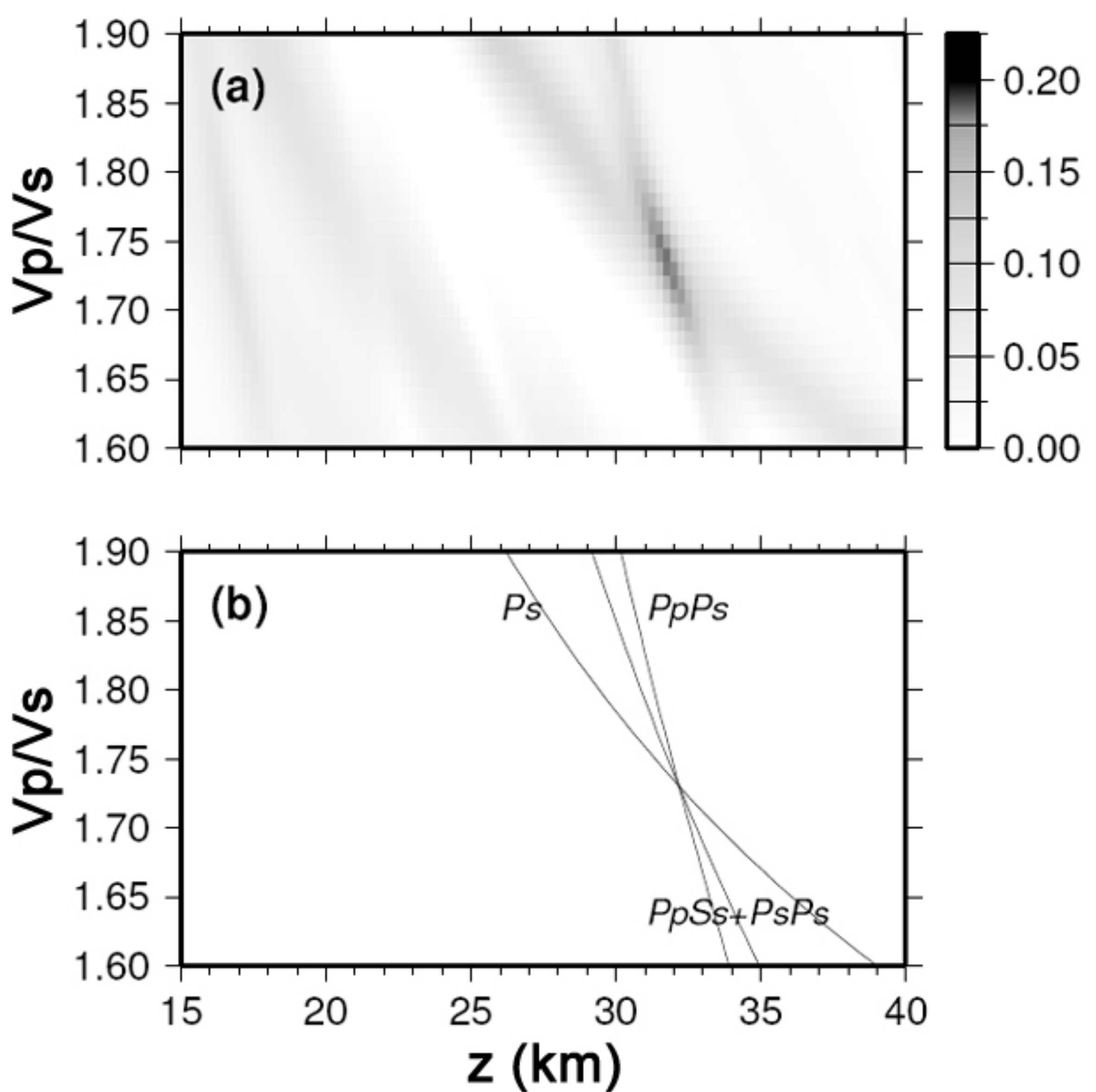
keep Vp constant and vary Vp/Vs, each system has a different behavior

				z_{Ps}	z_{PpPs}	$z_{PsPs+PpSs}$
Vp (km/s)	**K (Vp/Vs)**	**Vs (km)**	**p (s/km)**	**H (km)**	**H (km)**	**H (km)**
6.4	1.70	3.76	0.06	43.7	39.7	40.6
6.4	1.72	3.72	0.06	42.5	39.4	40.1
6.4	1.74	3.68	0.06	41.4	39.1	39.6
6.4	1.76	3.64	0.06	40.3	38.8	39.1
6.4	1.78	3.60	0.06	39.3	38.5	38.7
6.4	1.80	3.56	0.06	38.3	38.2	38.2
6.4	1.82	3.52	0.06	37.4	37.9	37.8
6.4	1.84	3.48	0.06	36.5	37.6	37.3
6.4	1.86	3.44	0.06	35.7	37.3	36.9
6.4	1.88	3.40	0.06	34.9	37.1	36.5
6.4	1.90	3.37	0.06	34.1	36.8	36.1
6.4	1.92	3.33	0.06	33.4	36.5	35.7
6.4	1.94	3.30	0.06	32.7	36.2	35.3
6.4	1.96	3.27	0.06	32.0	36.0	35.0
6.4	1.98	3.23	0.06	31.4	35.7	34.6

Only 1 Vp/Vs value leads the travel-times to a unique convergence on a certain thickness.

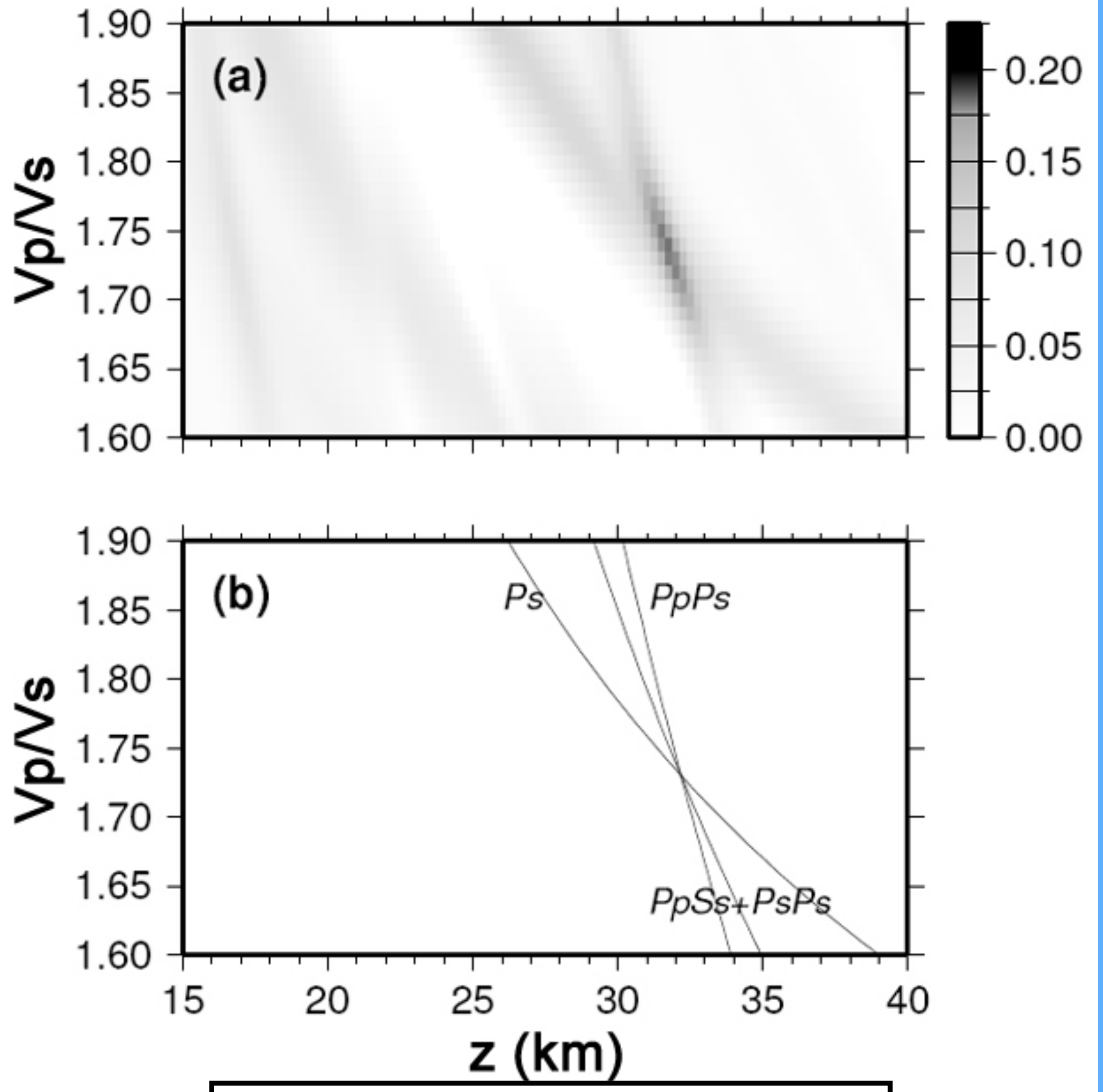






Zhu and Kanamori, 2000

6-12 months of events needed for good solution



Zhu and Kanamori, 2000

W1 = 0.7

W2 = 0.2

W3 = 0.1

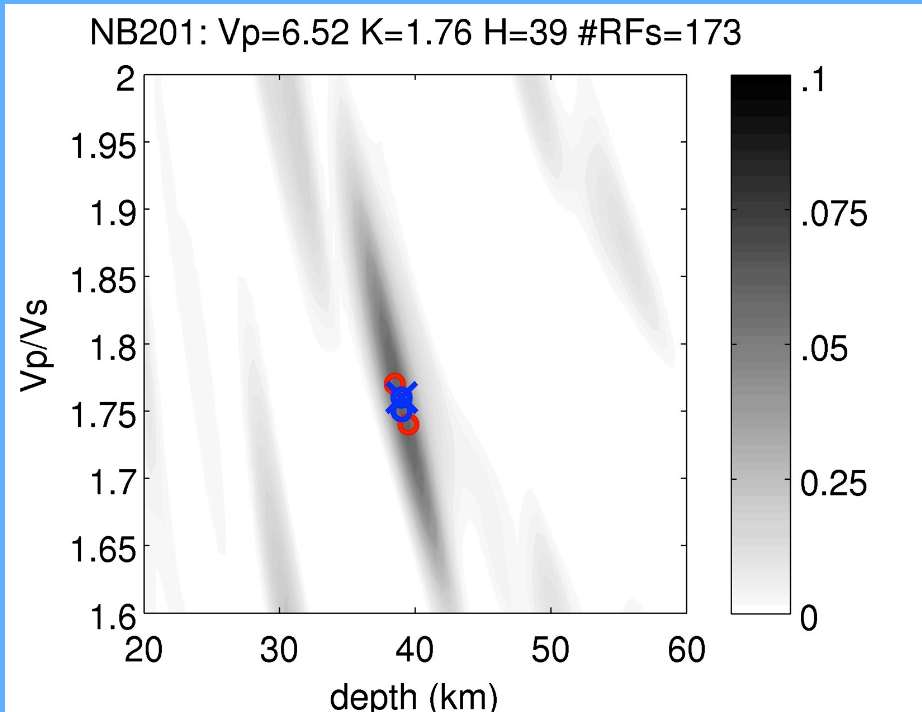
or

W1 = 0.34

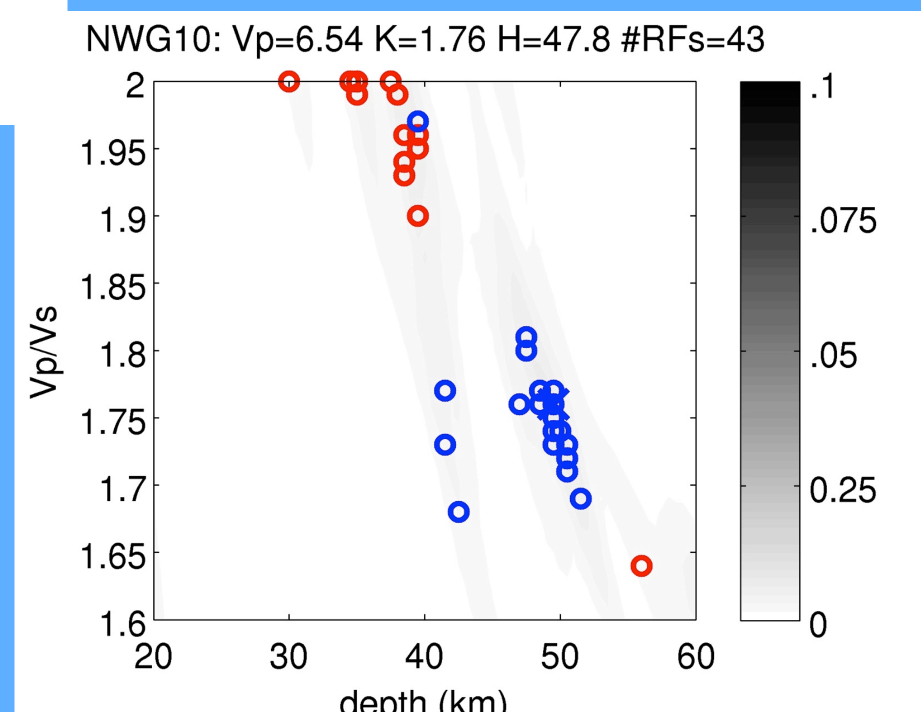
W2 = 0.33

W3 = 0.33

$$S(z, V_P / V_S) = w_1 r(t_1) + w_2 r(t_2) - w_3 r(t_3)$$


NB201: Vp=6.52 K=1.76 H=39 #RFs=173
Vp/Vs
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
1.6
20
30
40
50
60
depth (km)
.1
.075
.05
0.25
0

GOOD


NWG10: Vp=6.54 K=1.76 H=47.8 #RFs=43
Vp/Vs
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
1.6
20
30
40
50
60
depth (km)
.1
.075
.05
0.25
0

BAD

A Review: 3 Equations

$$z = \frac{t_{Ps}}{\sqrt{V_S^{-2} - p^2} - \sqrt{V_P^{-2} - p^2}}$$

We Know:

t and *p*

$$z = \frac{t_{PpPs}}{\sqrt{V_S^{-2} - p^2} + \sqrt{V_P^{-2} - p^2}}$$

We Assume:

Vp

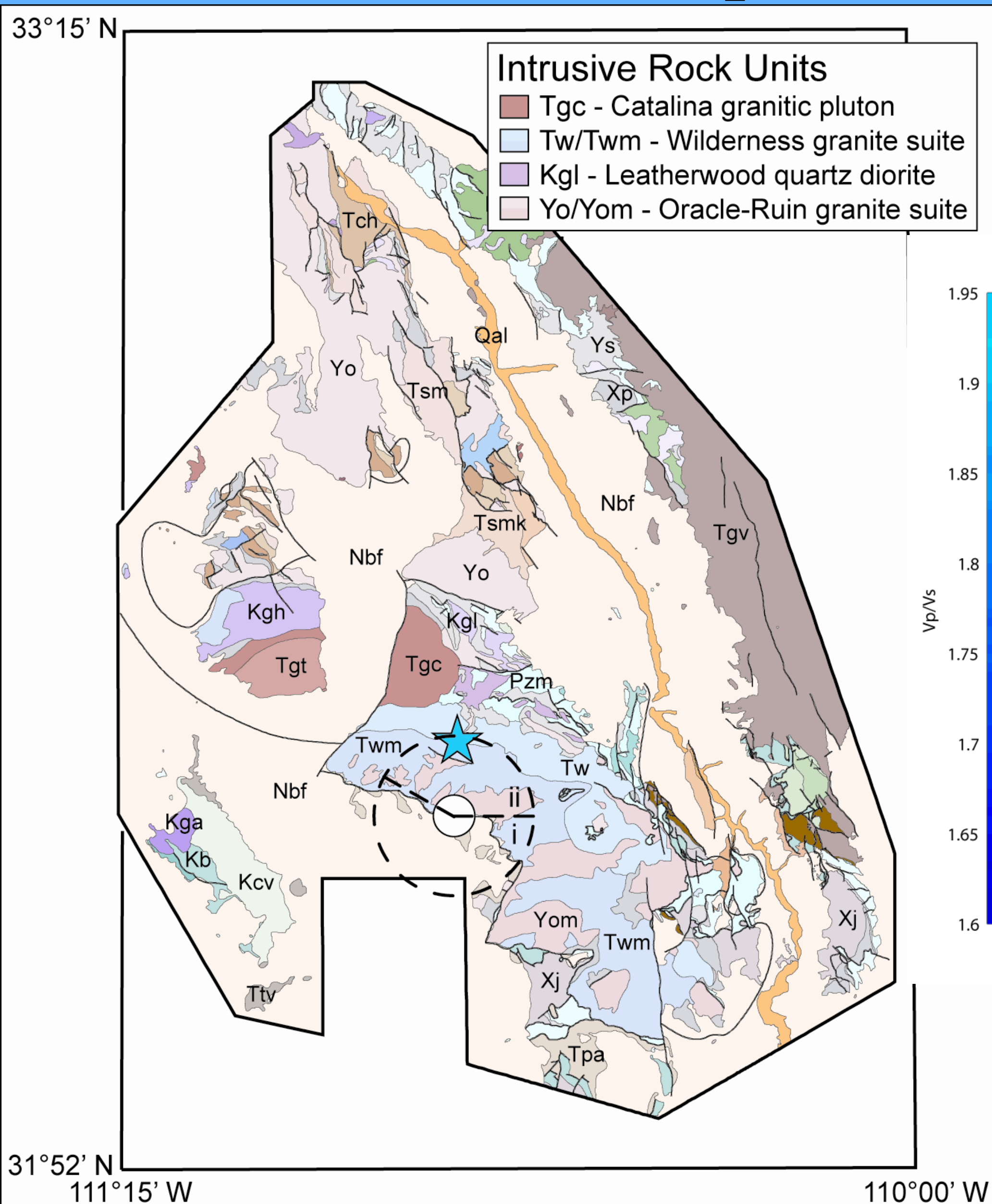
$$z = \frac{t_{PpSs+PsPs}}{2\sqrt{V_S^{-2} - p^2}}$$

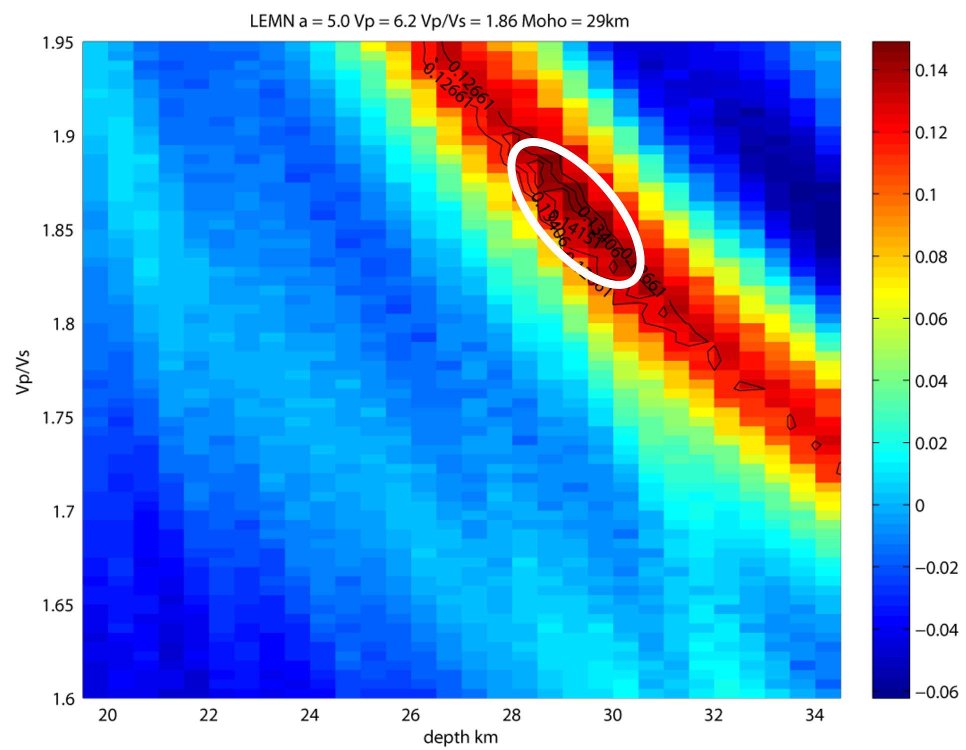
We Calculate:

z and Vs (giving us Vp/Vs)

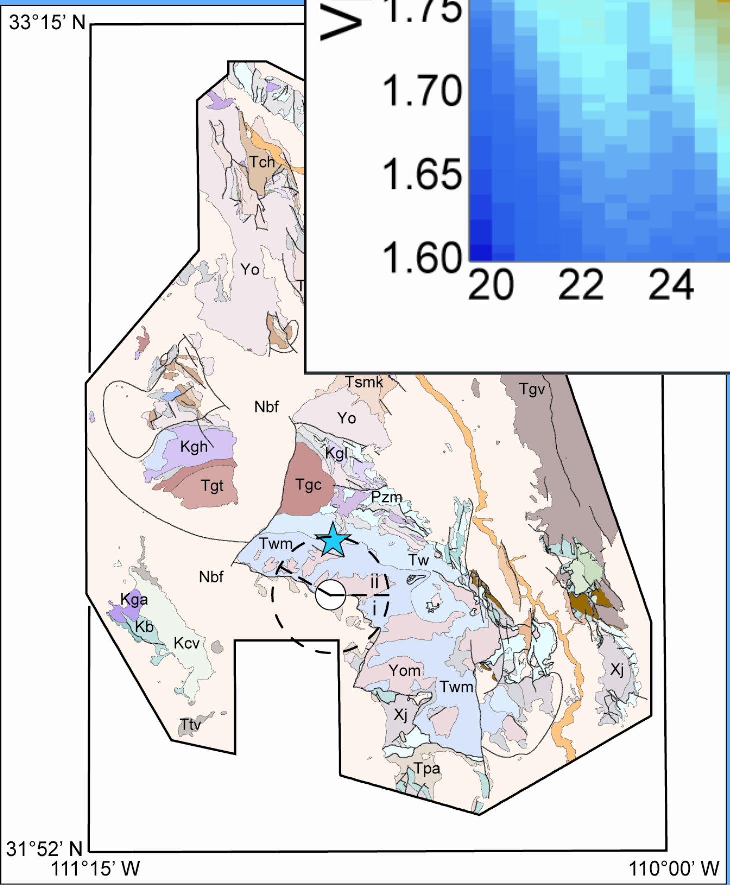
Example: Arizona

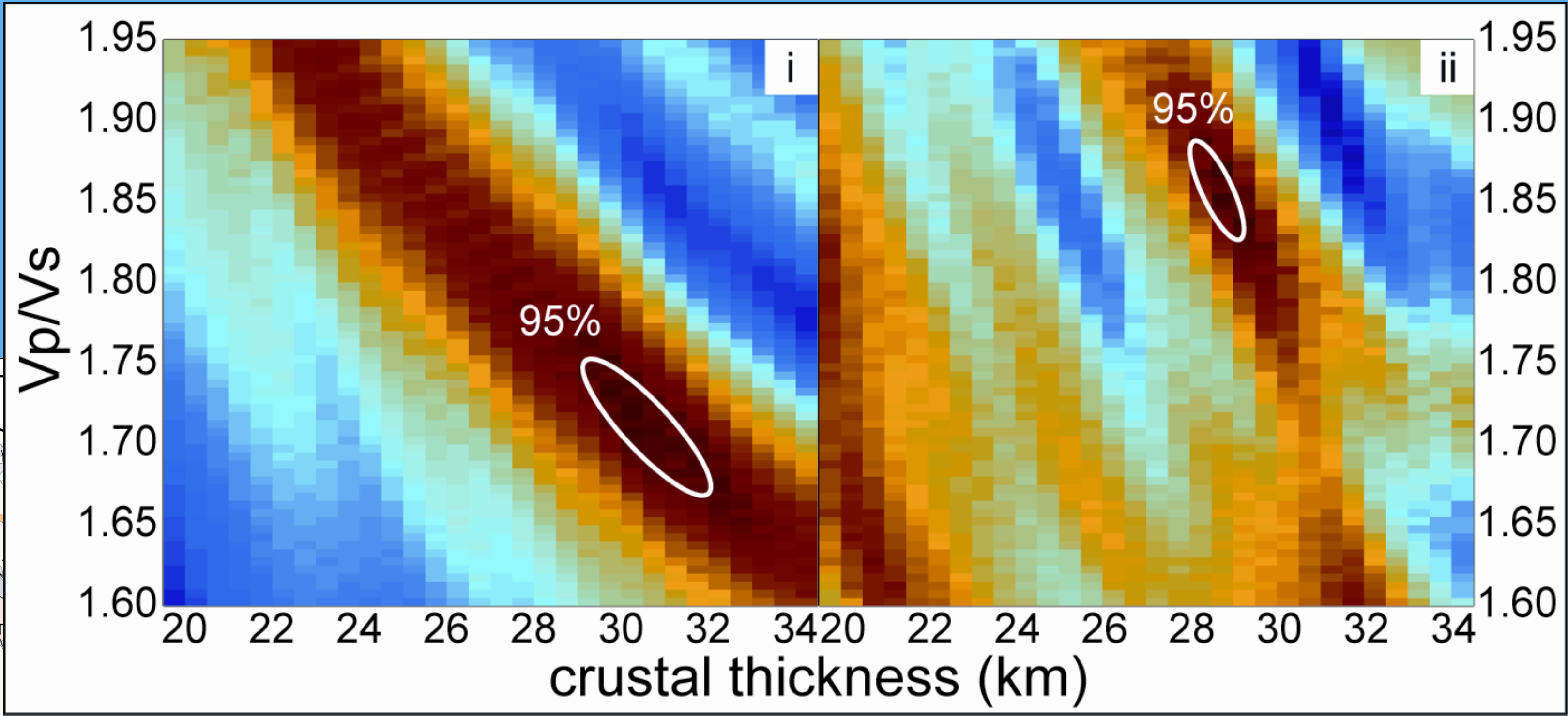
Example: Arizona



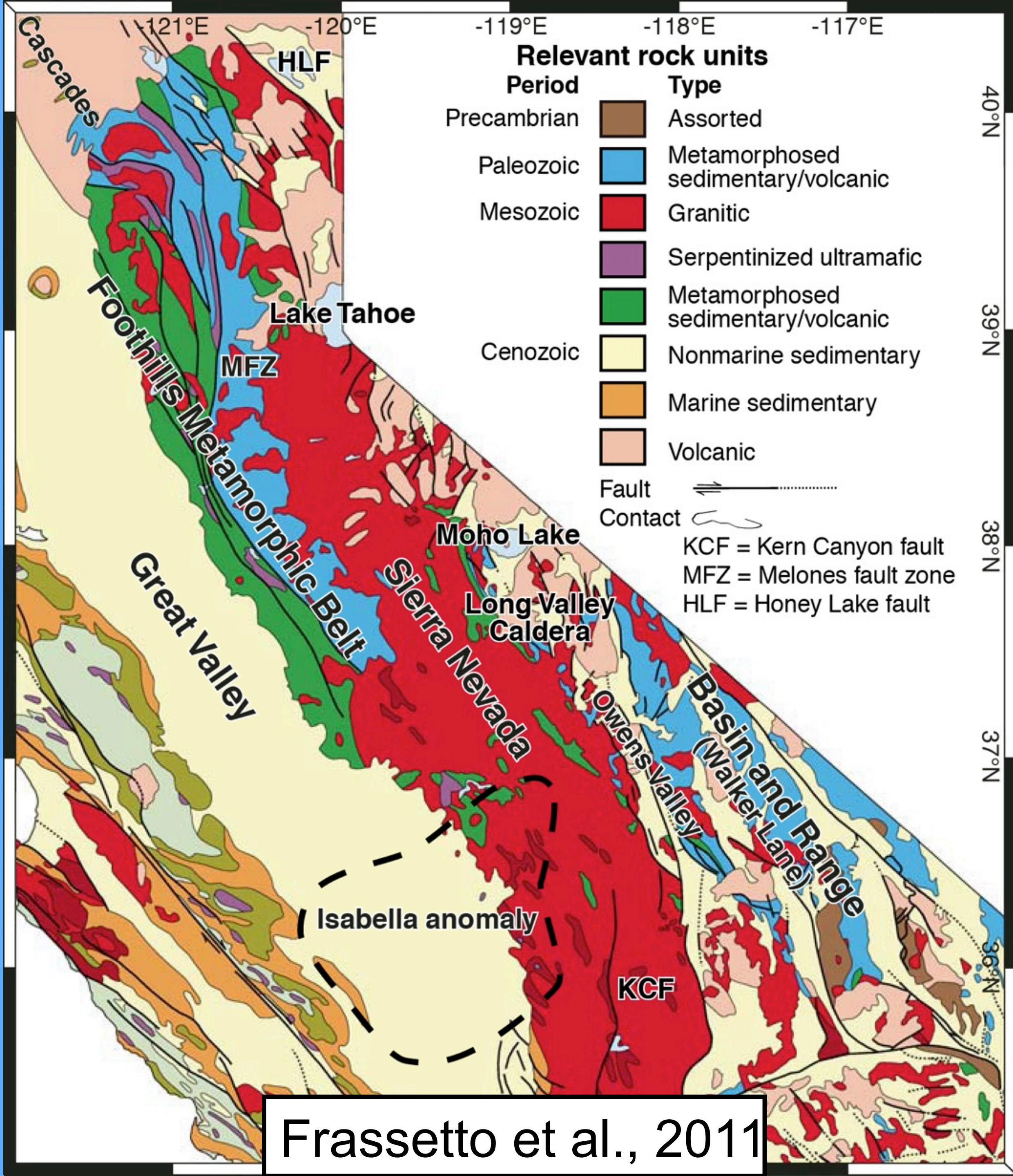


Example: Arizona



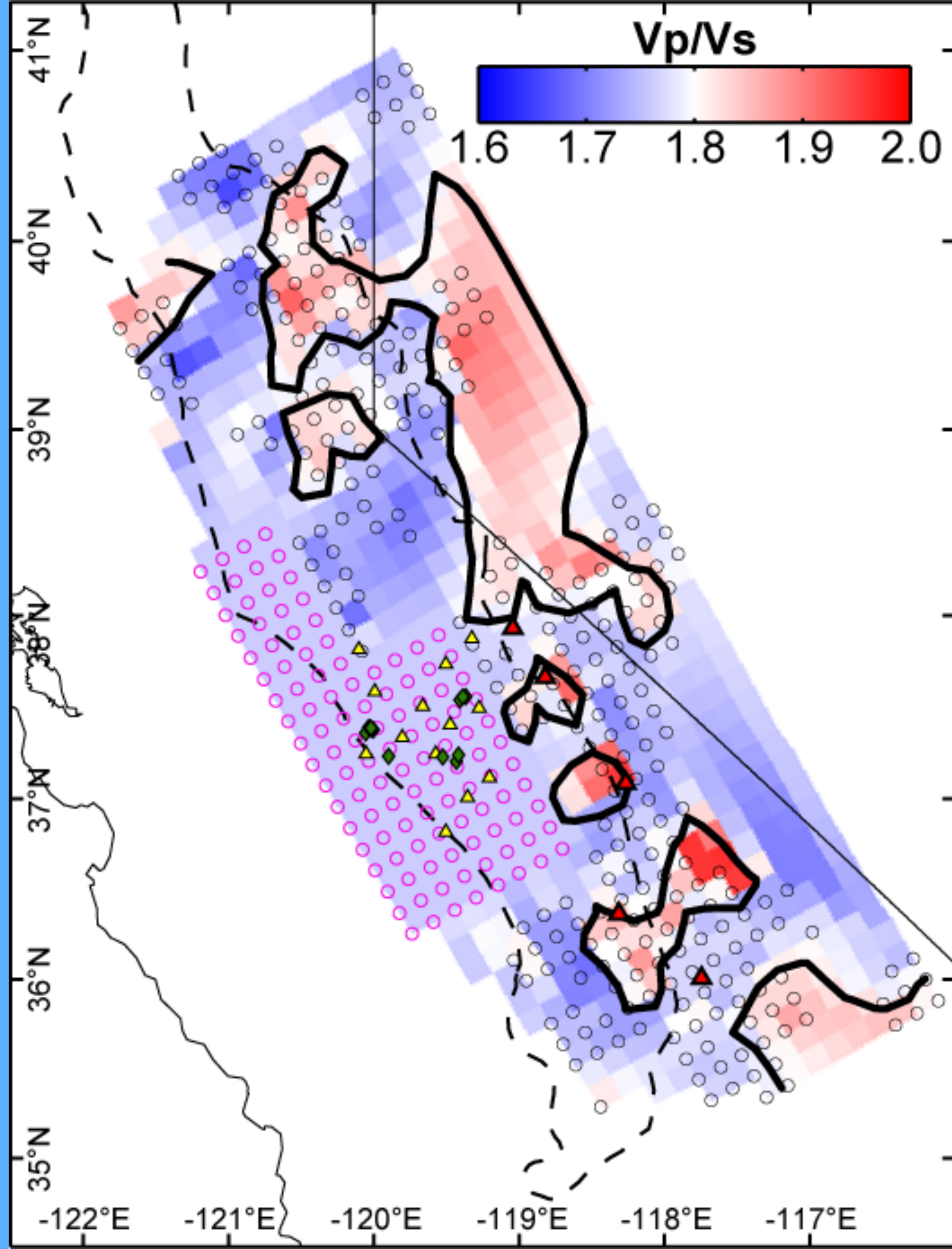


Example: Californi a

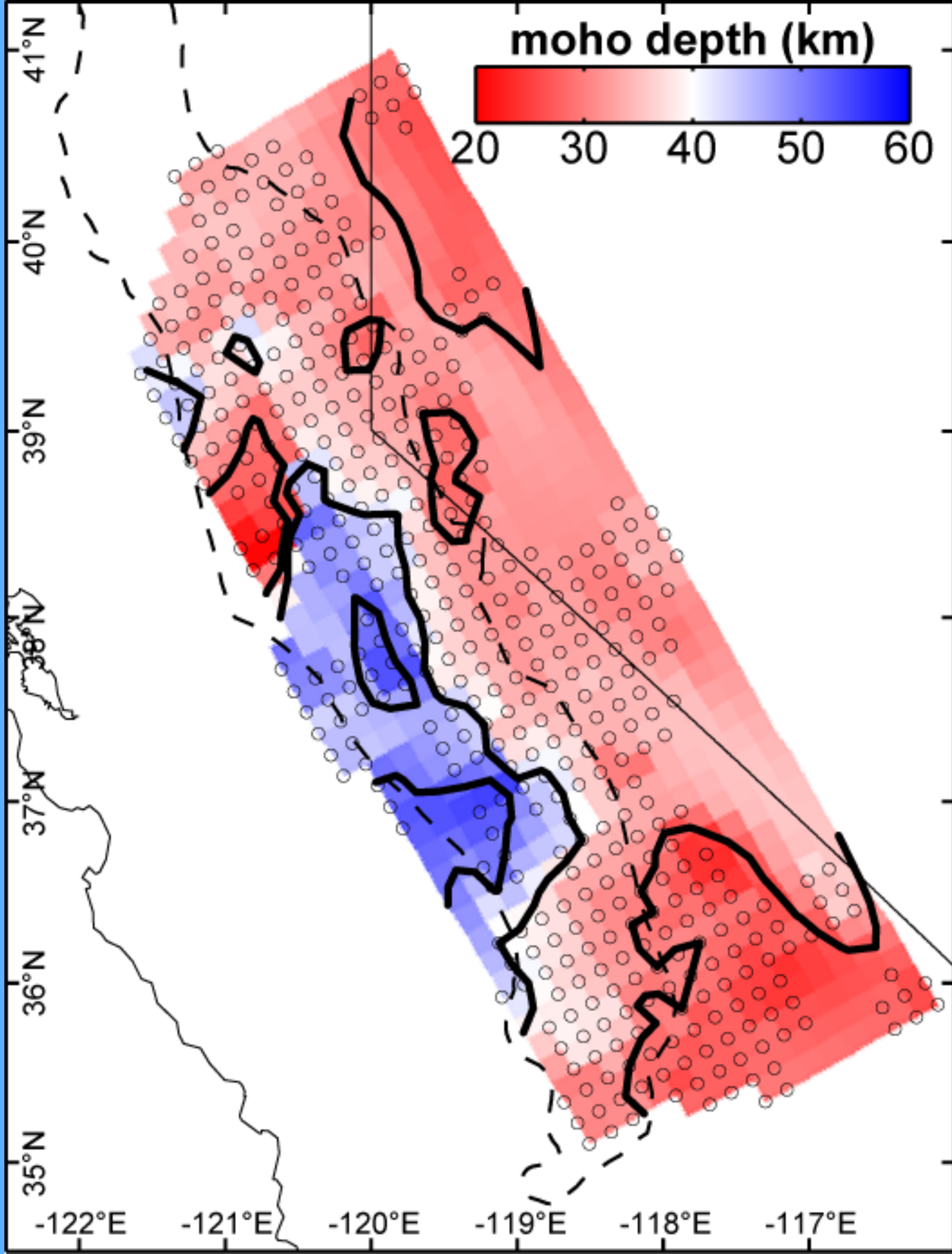


Frassetto et al., 2011

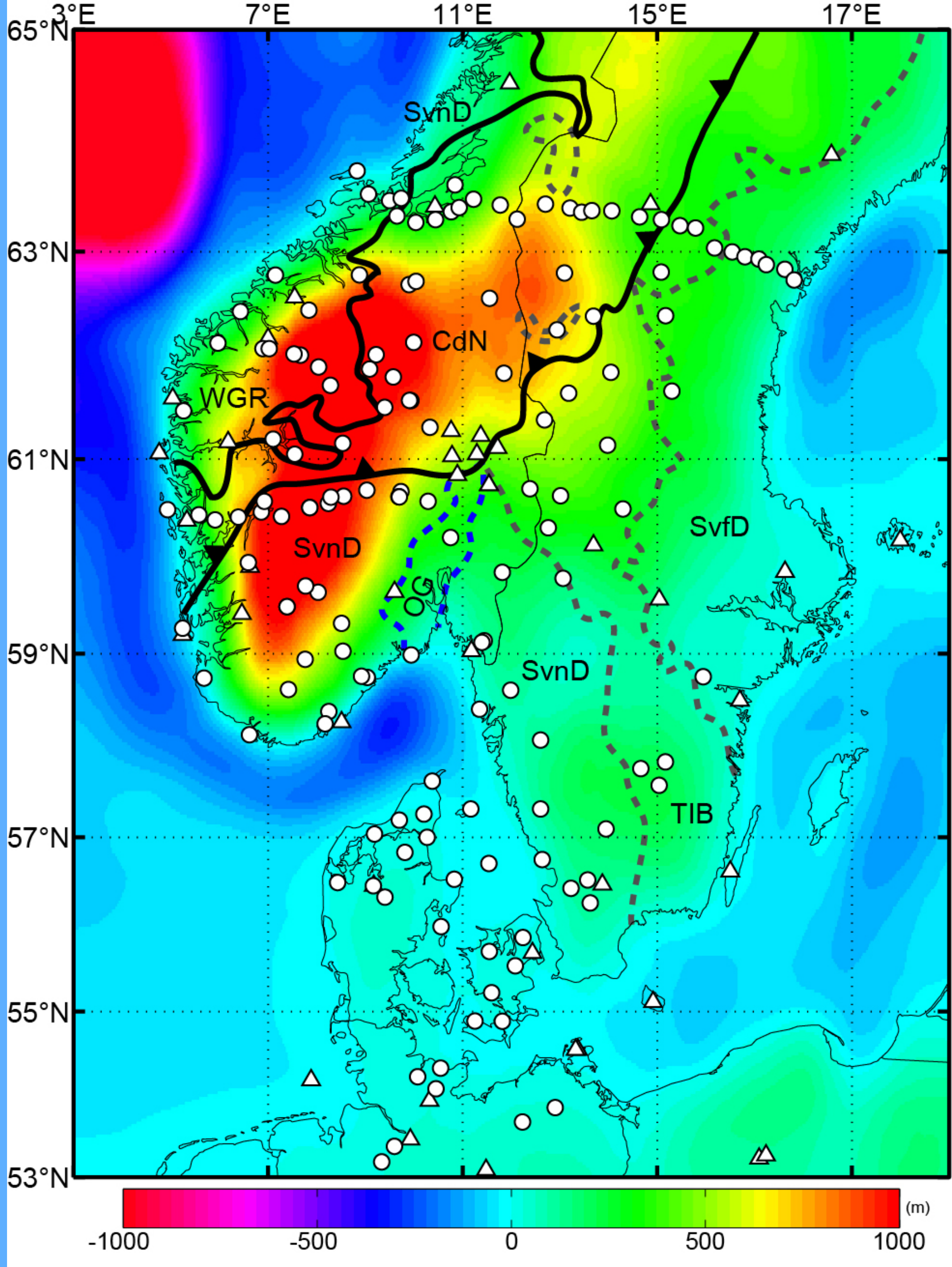
Example: California



Example: Californi a

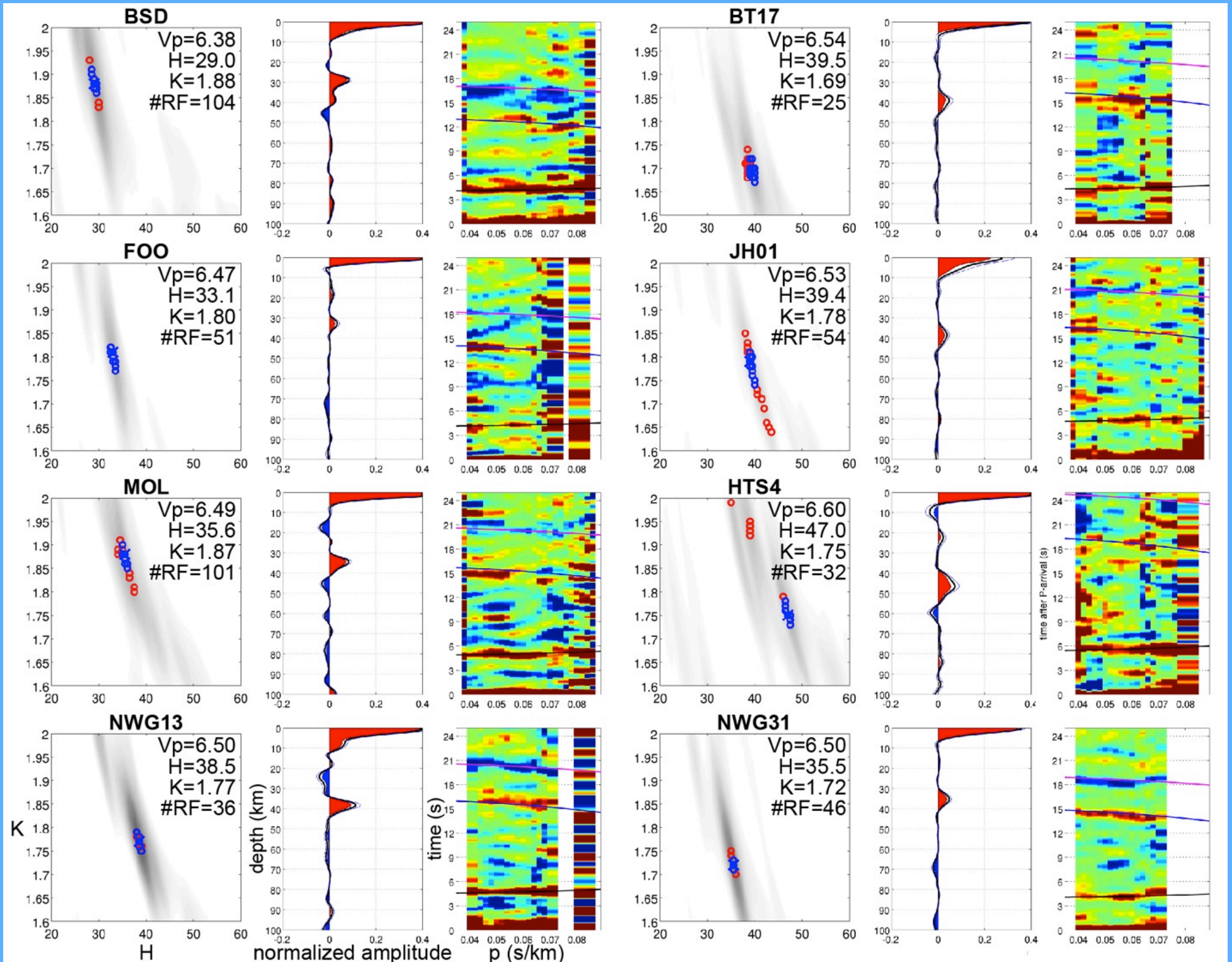


Example: Norway

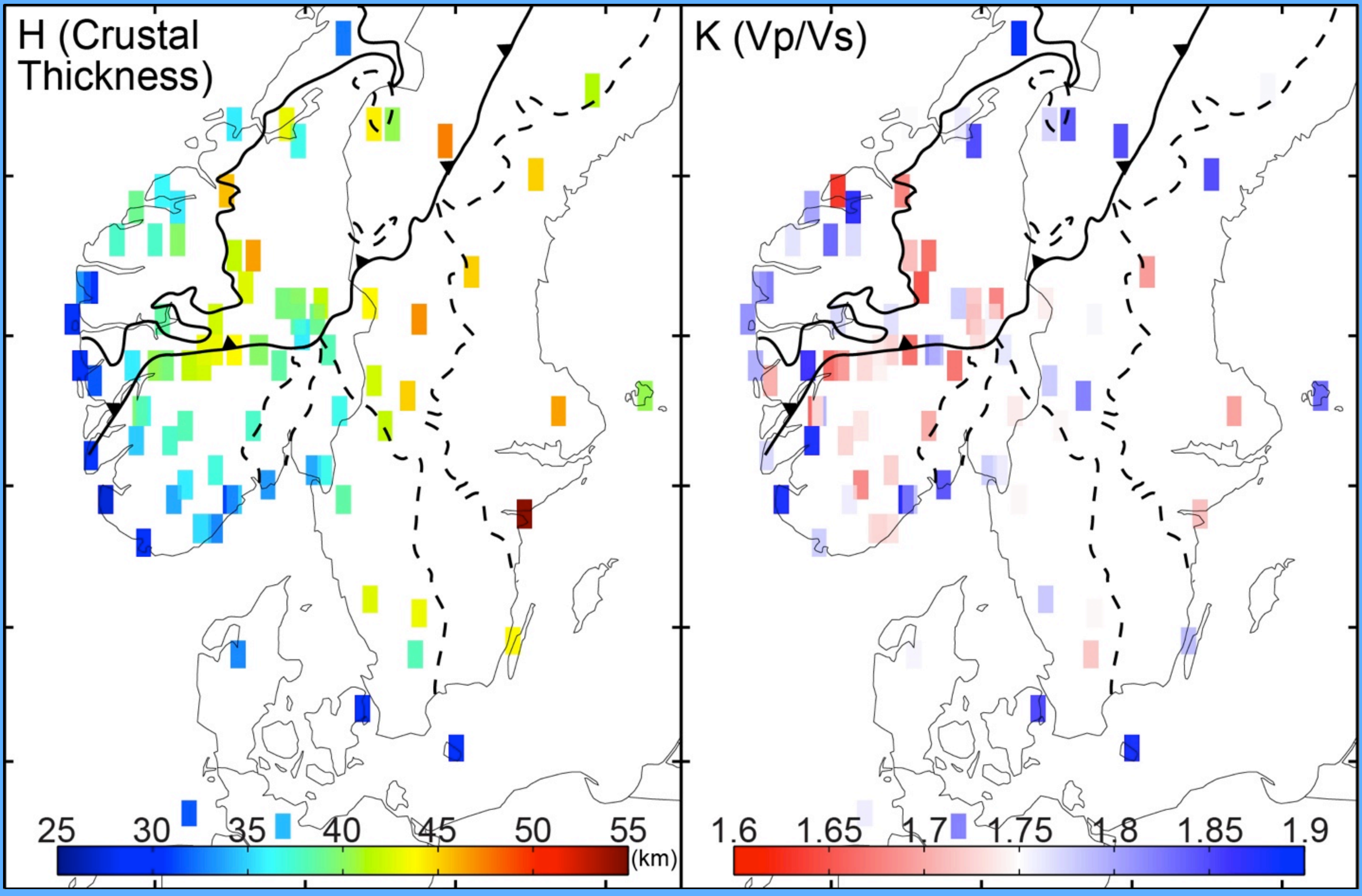


Frassetto et al., Submitted

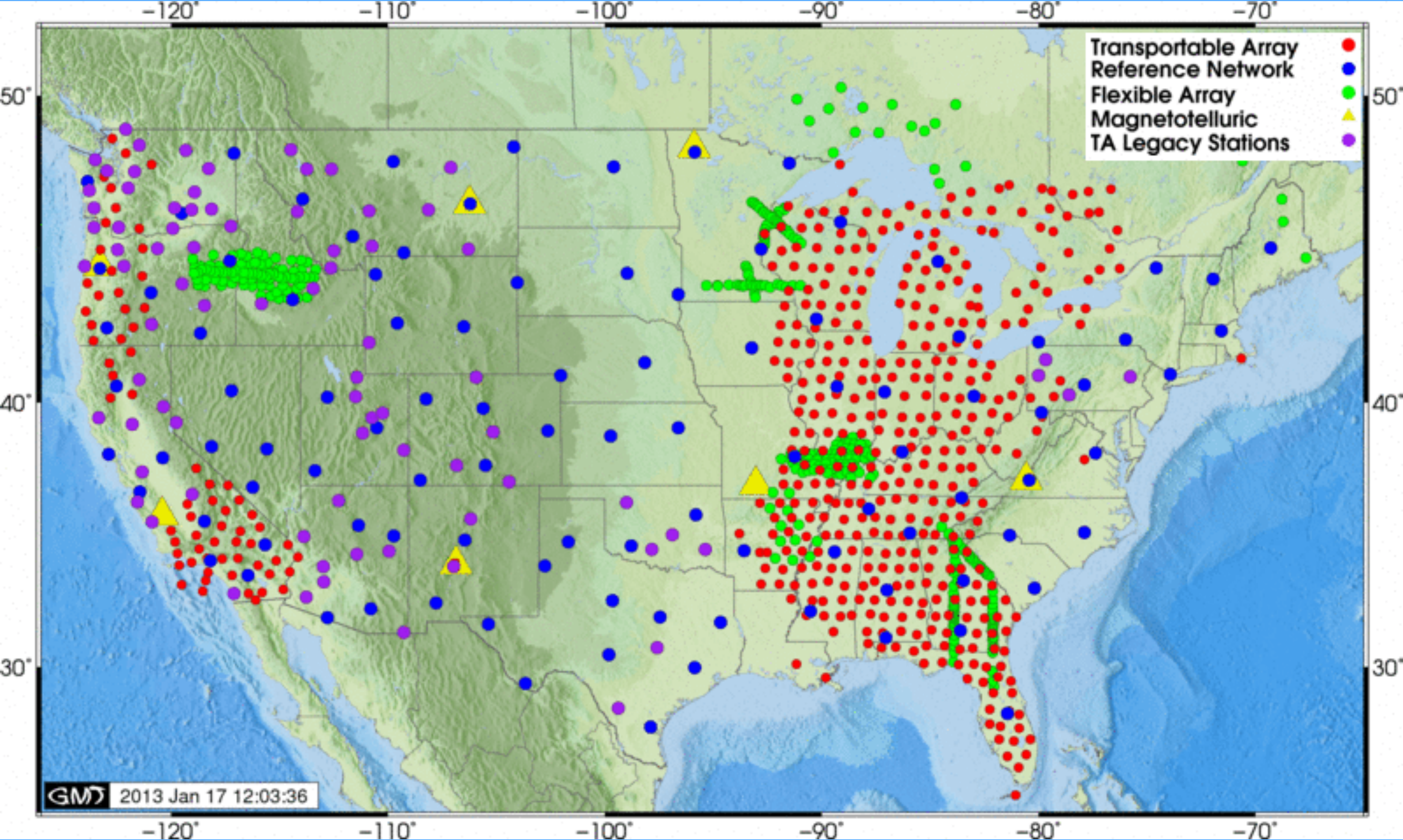
Example: Norway

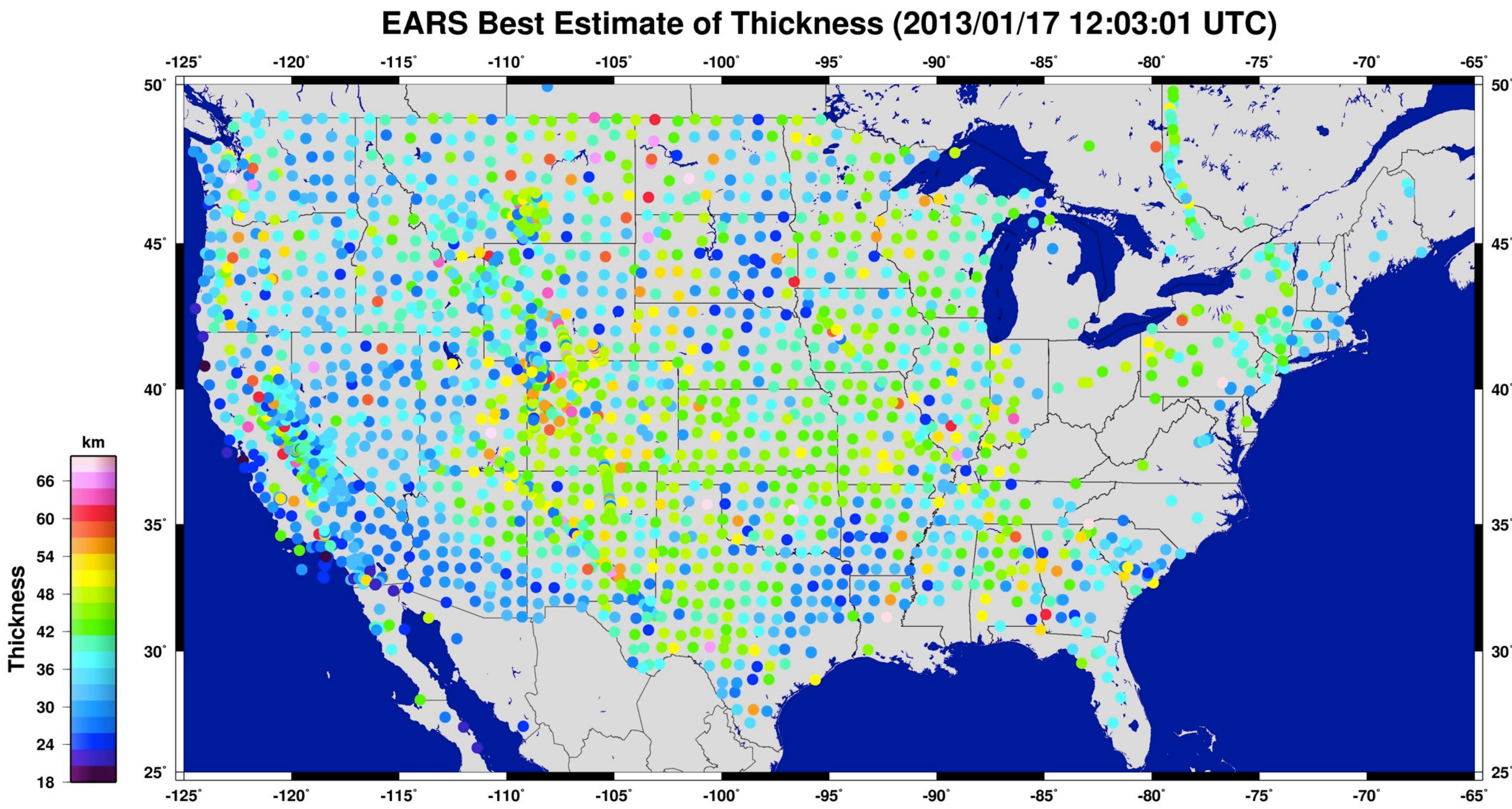


Example: Norway



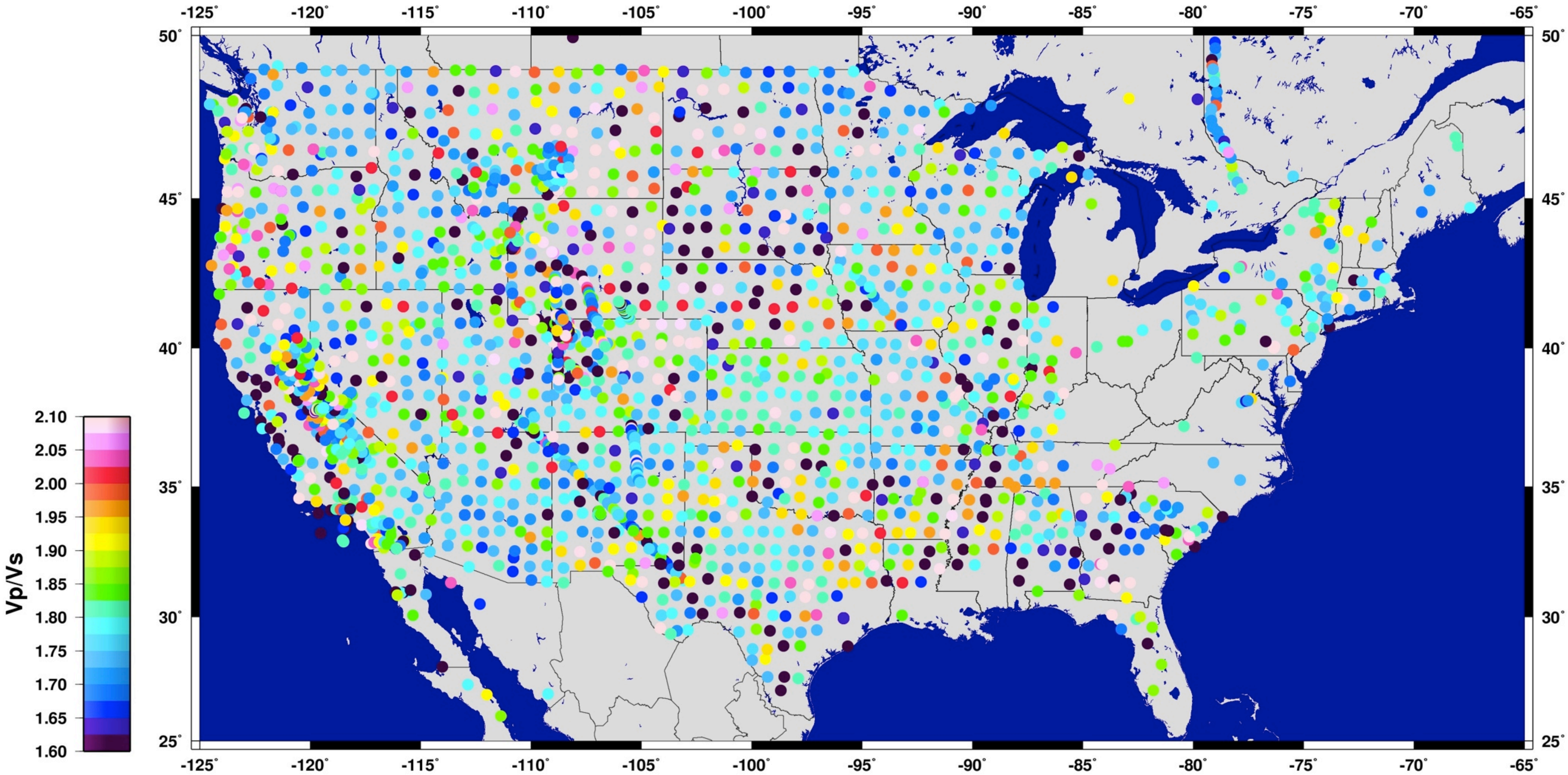
Example: USArray



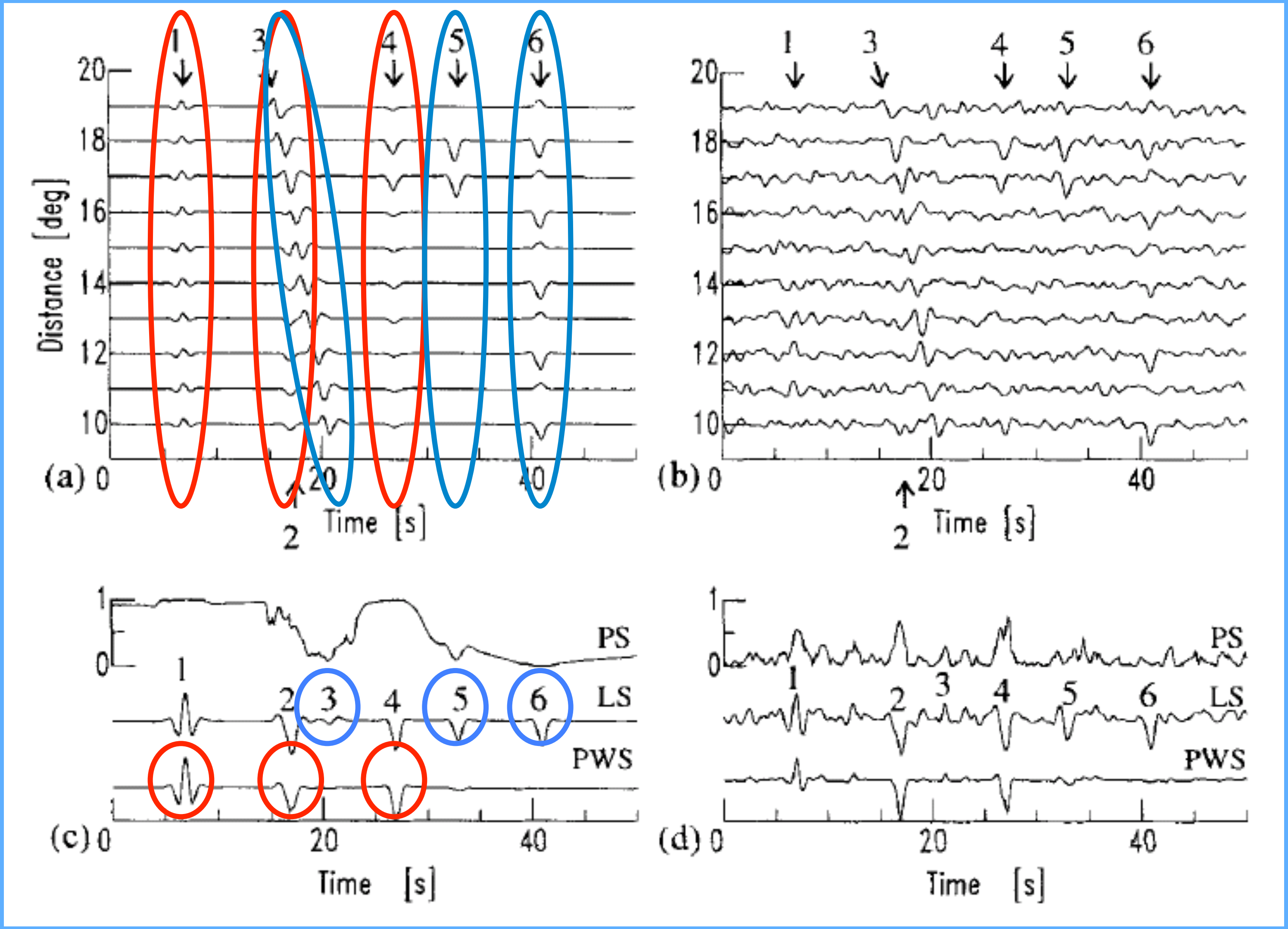


http://www.iris.edu/dms/products/ears/

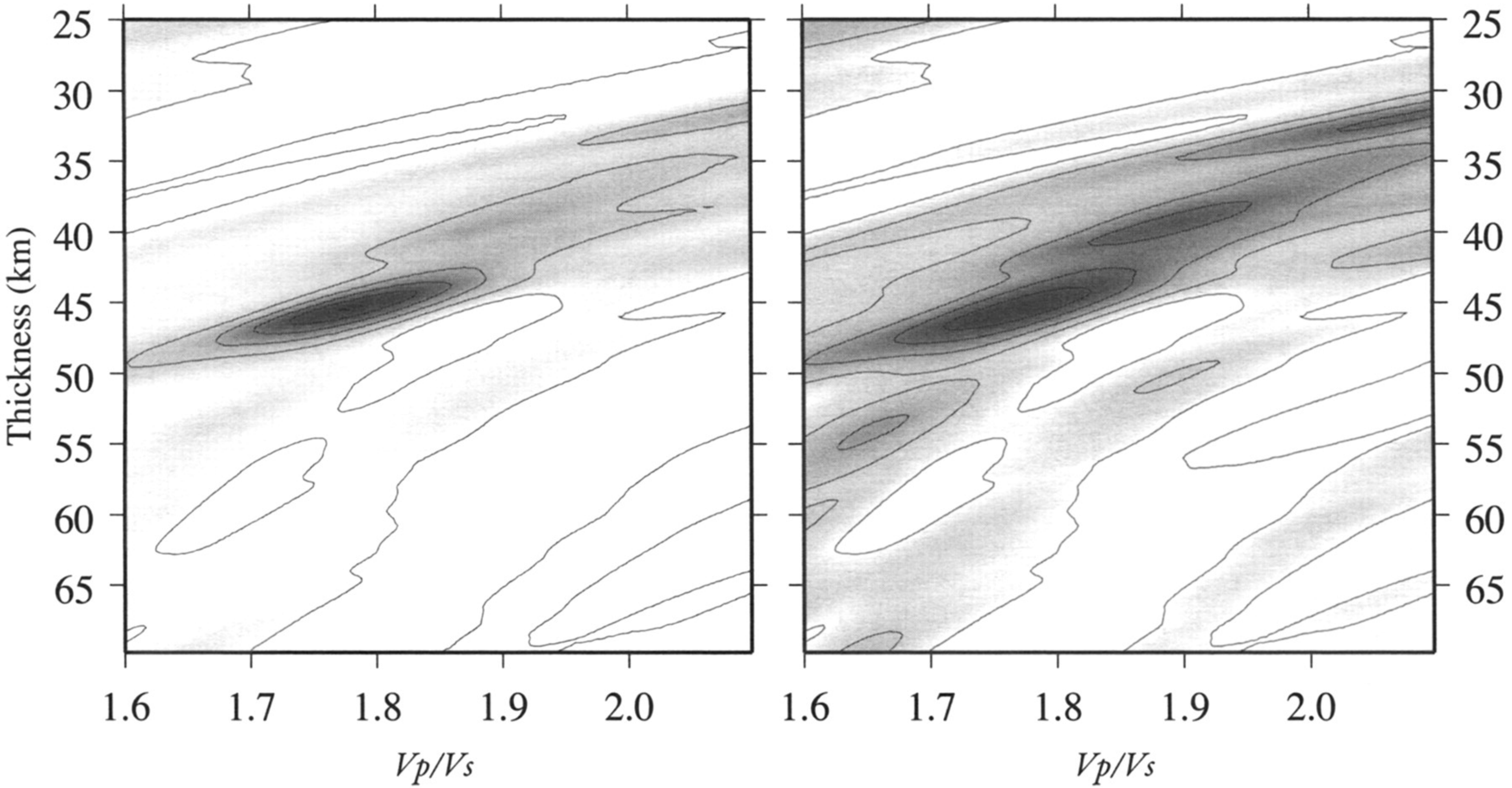
EARS Best Estimate of Vp/Vs (2013/01/17 12:03:01 UTC)



Phase Weighting

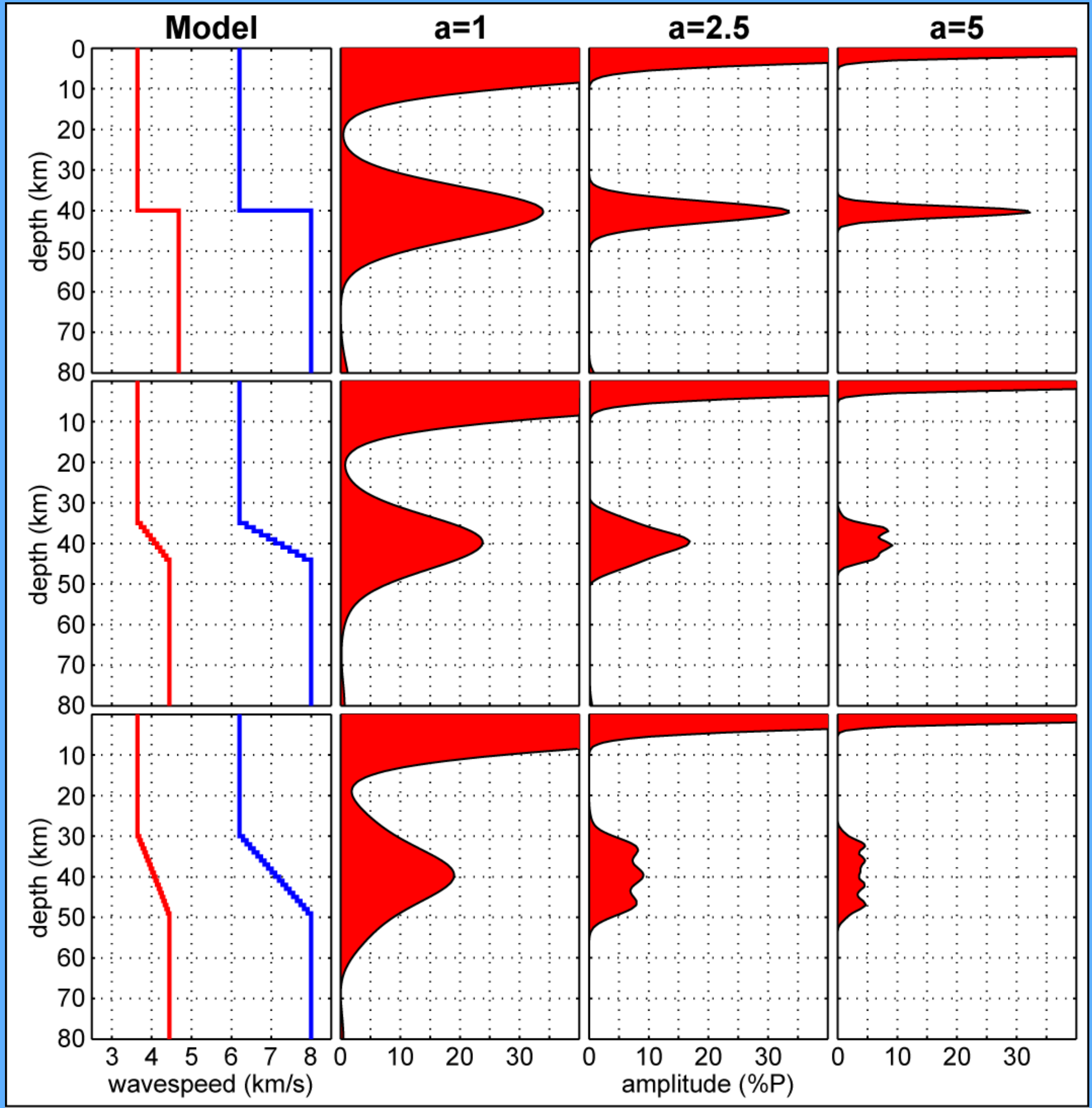


Schimmel and Paulssen, 1997

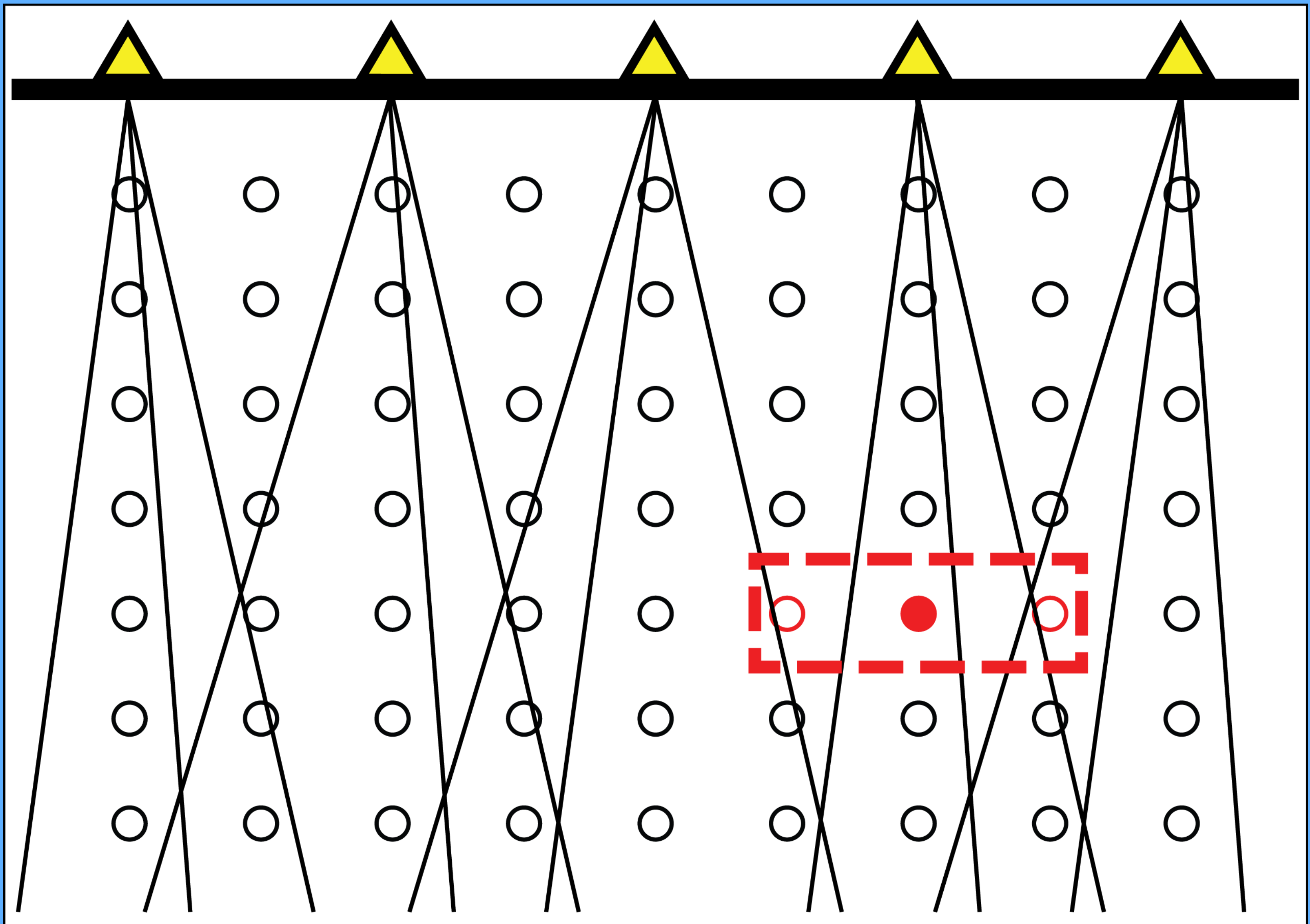


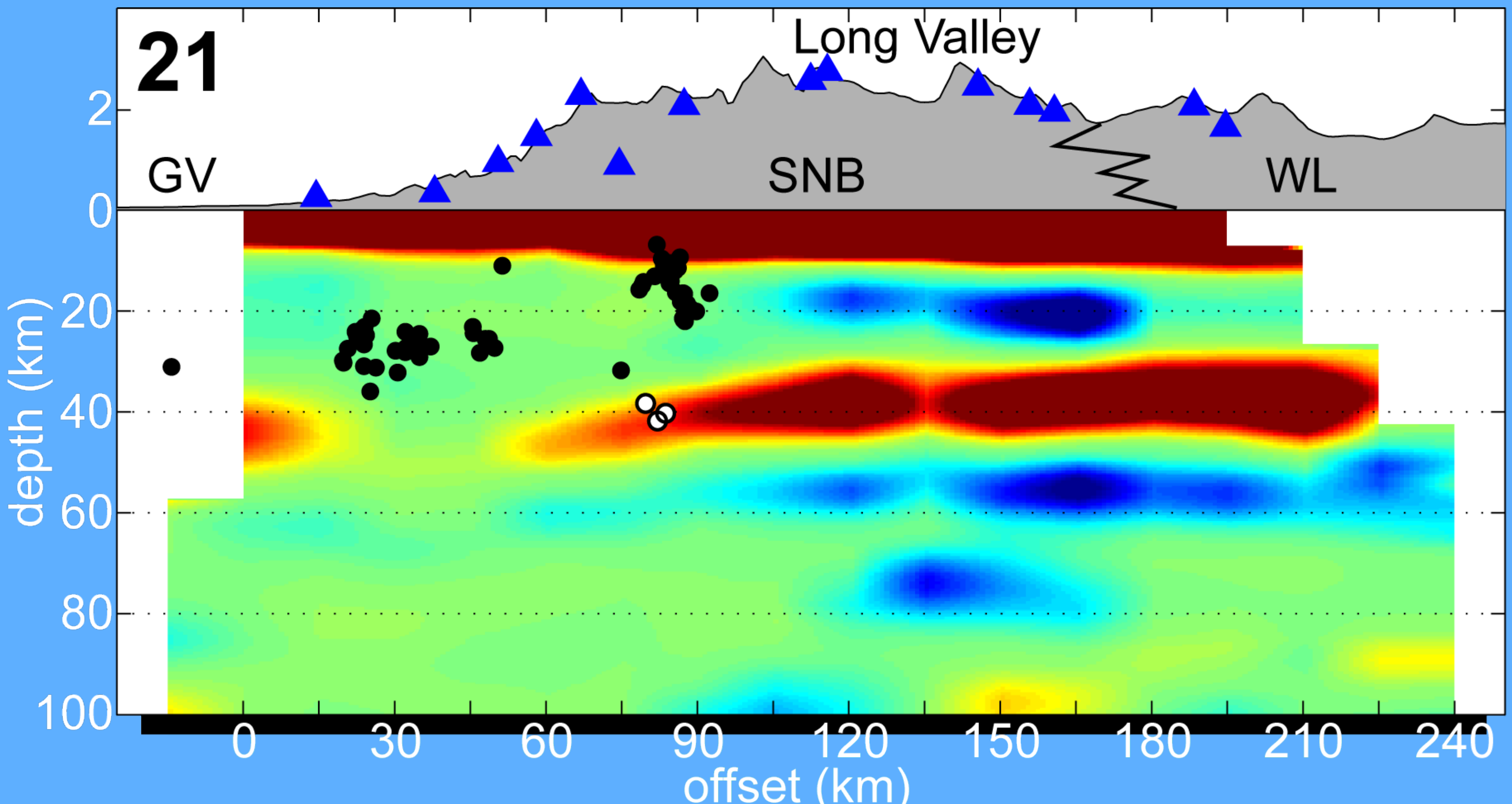
Crotwell and Owens, 2005

Interface Resolution

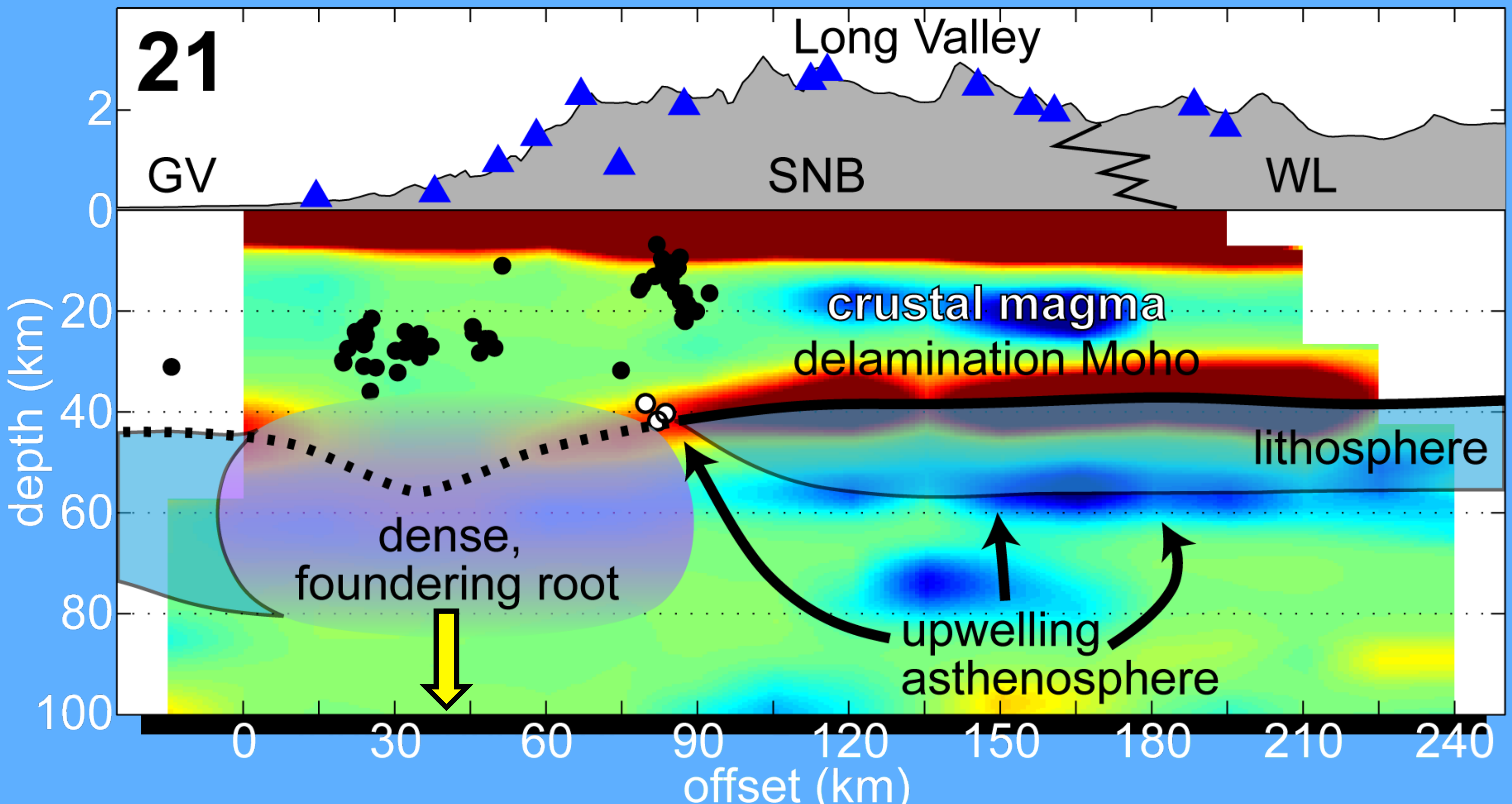


CCP Stacking





21
Long Valley
GV
SNB
WL
depth (km)
offset (km)
0
2
20
40
60
80
100
0
30
60
90
120
150
180
210
240



21
Long Valley
GV
SNB
WL
2
0
20
40
60
80
100
depth (km)
crustal magma
delamination Moho
lithosphere
dense,
foundering root
upwelling
asthenosphere
0
30
60
90
120
150
180
210
240
offset (km)


Questions?