Linearized Joint Inversion

Advanced Studies Institute on Seismological Research

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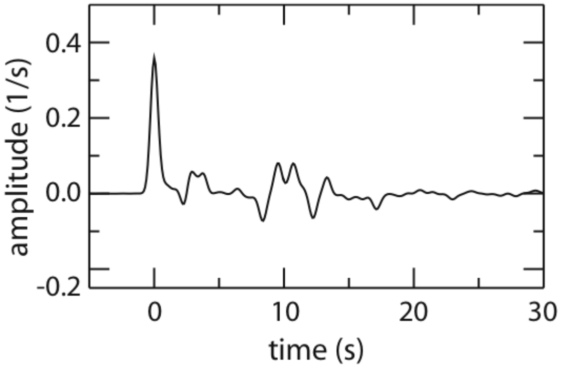
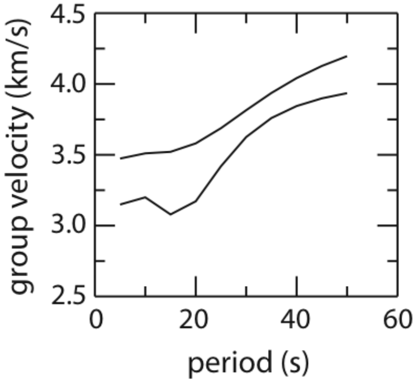
Outline

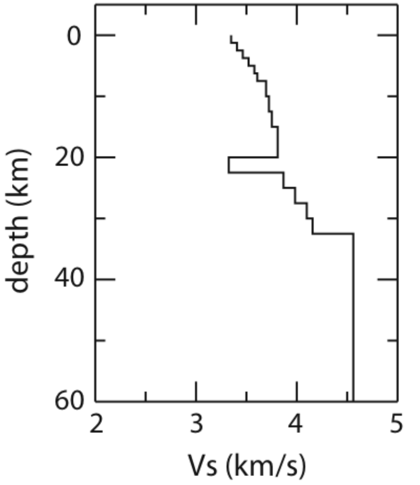
- Joint Inversion of PRFs and SW:
  - Why a joint inversion?
  - Method of Julià et al. (2000)
- Case study in Brazil:
  - The Paraná basin (Julià et al., 2008)

Why a Joint Inversion?

- We have already seen that receiver functions are sensitive to S-wave velocities.
- And we have also seen that the inversion is non-unique.
- On the other hand, we have just seen that dispersion velocities, like receiver functions, are sensitive to S-velocities.
- Is it possible to find a single model that can simultaneously fit BOTH data sets?

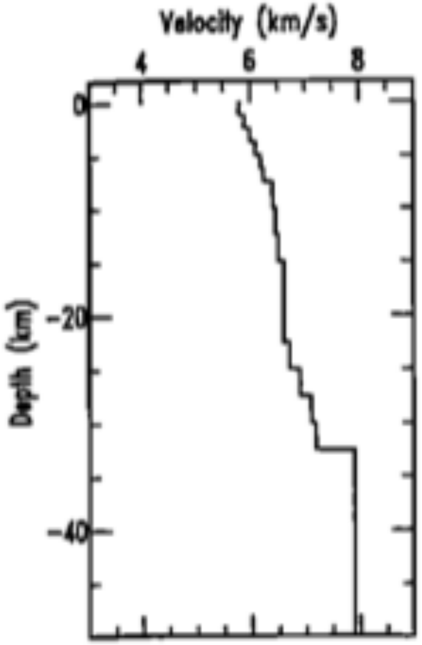






Receiver function ONLY inversion

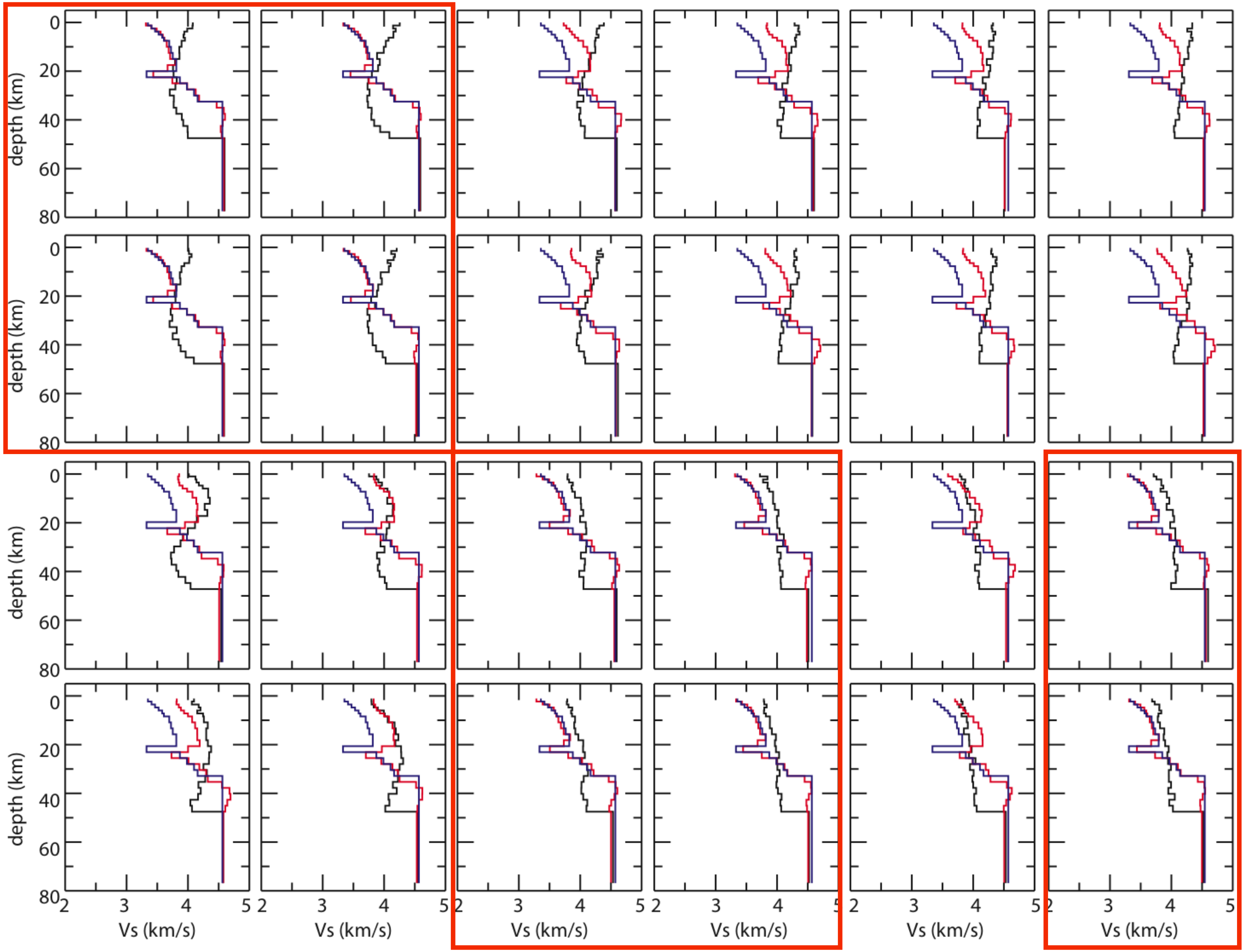
Velocity models are over-parameterized through a stack of many thin layers of constant thickness and unknown S-velocity. A smoothness constrain is needed to stabilize the inversion.

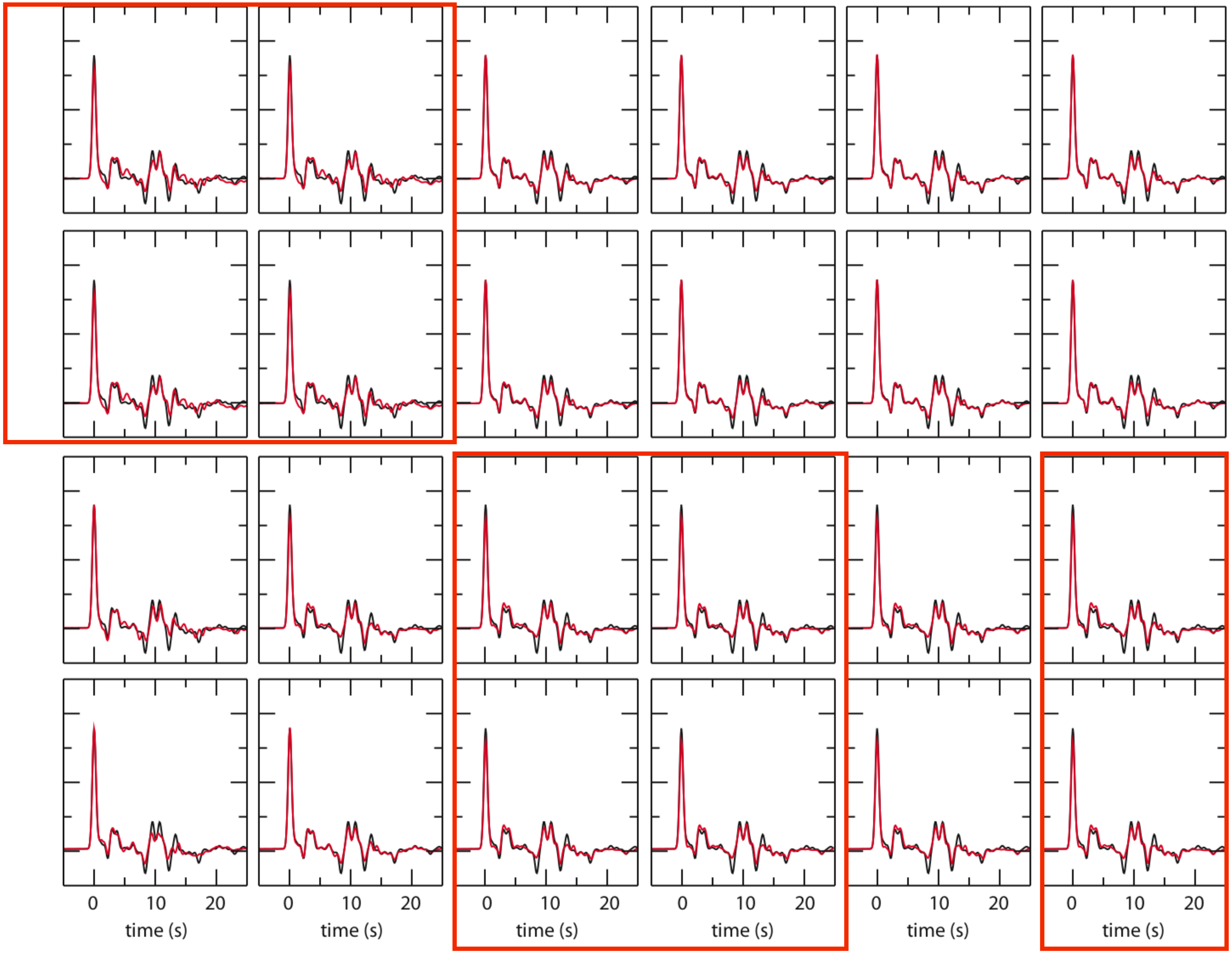


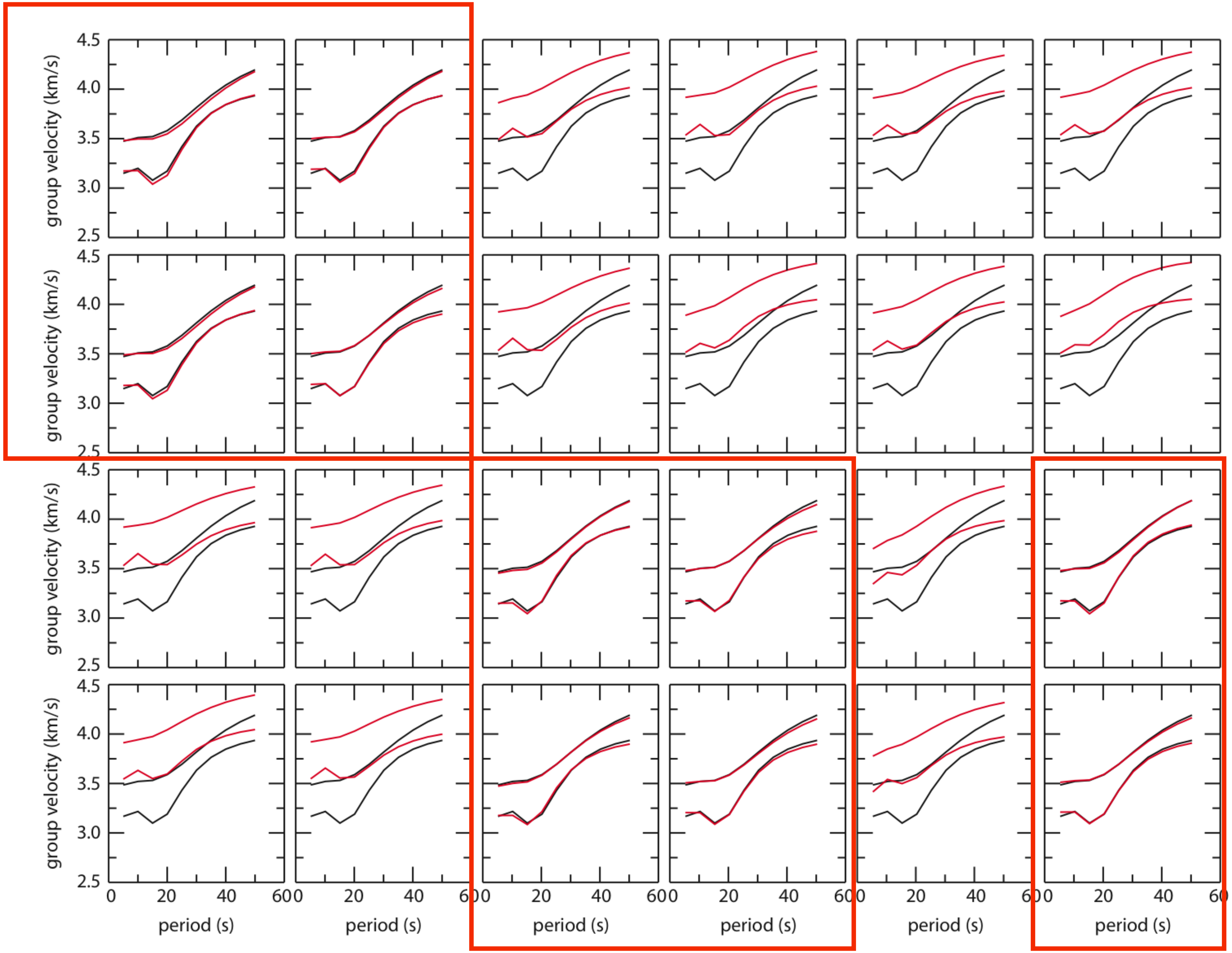
$$\begin{cases} \Delta \mathbf{d} + \nabla F\, \mathbf{m}_0 = \nabla F|_{\mathbf{m}0}\, \mathbf{m} \\ \mathbf{0} = \sigma\, D\, \mathbf{m} \end{cases}$$

$$D\,\mathbf{m} = \begin{bmatrix} 1 & -2 & 1 & \\ & 1 & -2 & 1 \\ & & \vdots & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \end{bmatrix}$$

$$E = \|\, \Delta \mathbf{d} - \nabla F\,(\mathbf{m}-\mathbf{m}_0)\,\|^2 + \sigma^2\, \|\, D\mathbf{m}\,\|^2$$

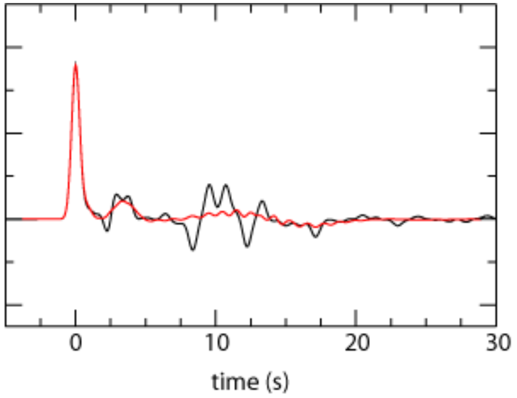

depth (km)
0
20
40
60
80
Vs (km/s)
2
3
4
5

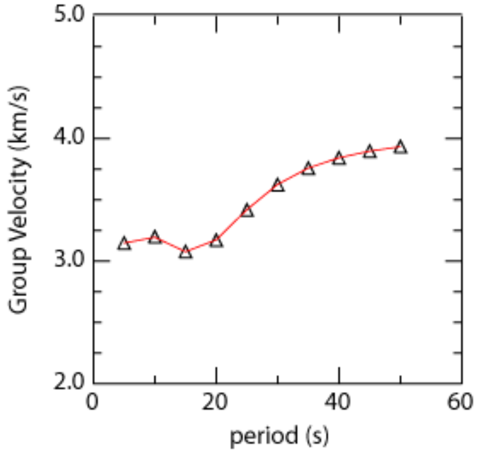

0
10
20
time (s)
0
10
20
time (s)
0
10
20
time (s)
0
10
20
time (s)
0
10
20
time (s)
0
10
20
time (s)

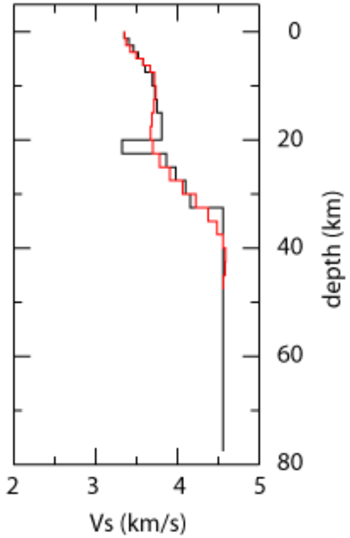

group velocity (km/s)
4.5
4.0
3.5
3.0
2.5
0
20
40
60
period (s)

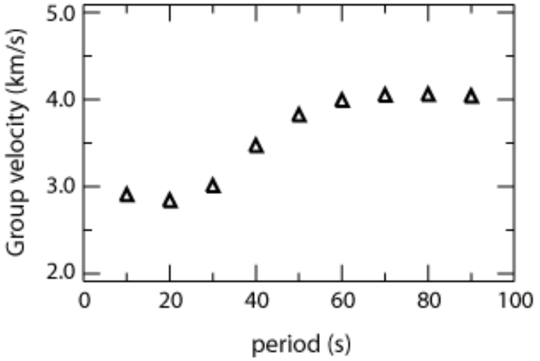
Dispersion ONLY inversion

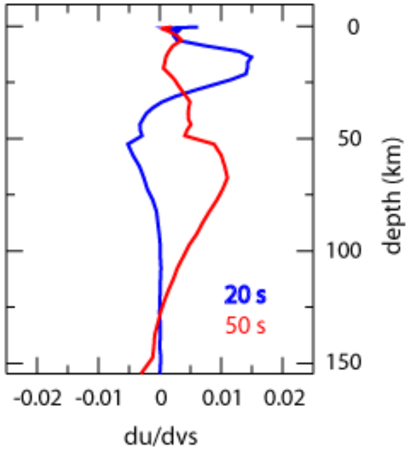
Inversion of dispersion velocities alone can constrain an average velocity models, but high-resolution details are missed out.







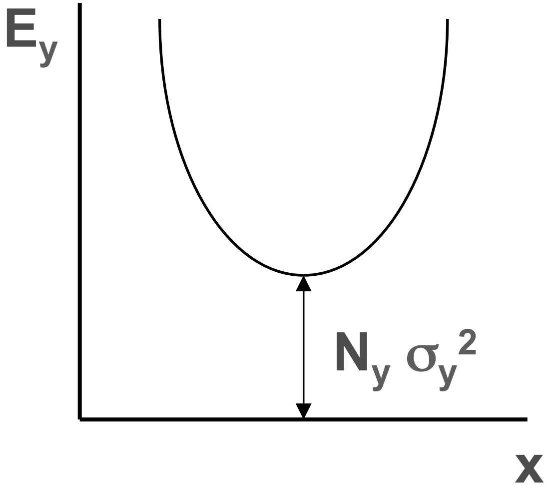


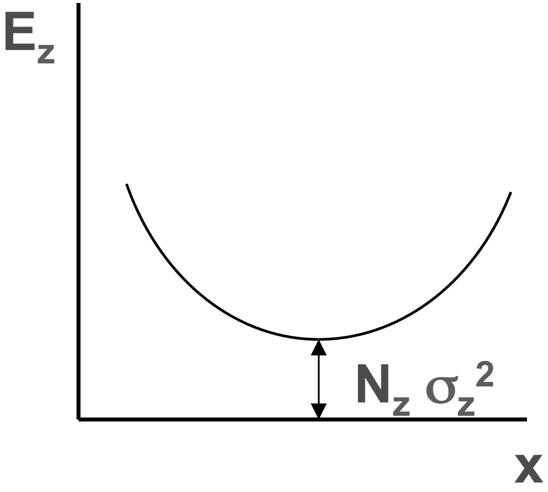


Inversion of Julià *et al.* (2000)

The problem we want to solve consists of inverting for two datasets that are sensitive to the same set of parameters.

$$\mathbf{y} = Y\,\mathbf{x} \qquad\qquad \mathbf{z} = Z\,\mathbf{x}$$

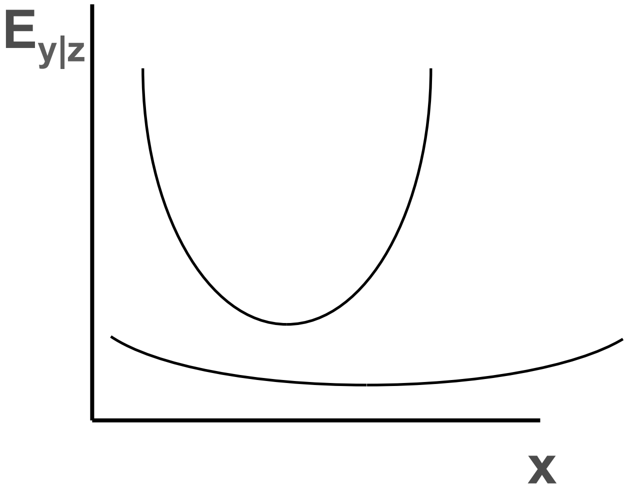


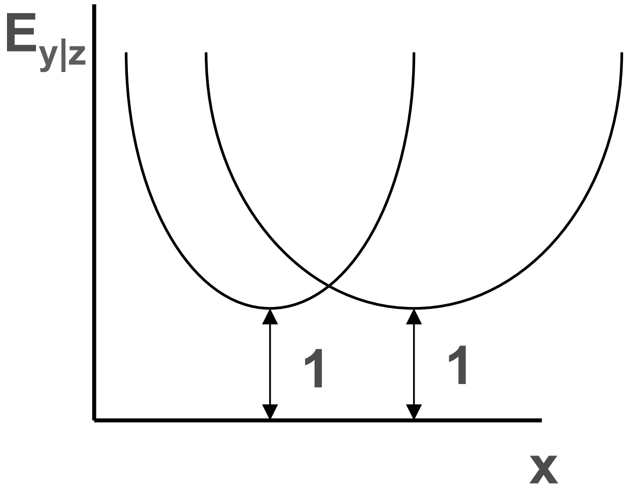


$$E_y = (\mathbf{y}\text{-}Y\mathbf{x})^T\,(\mathbf{y}\text{-}Y\mathbf{x}) \qquad\qquad E_z = (\mathbf{z}\text{-}Y\mathbf{x})^T\,(\mathbf{z}\text{-}Y\mathbf{x})$$

Equalizing the data sets

This cannot be achieved by simply minimizing the sum of the objective functions. We must first normalize to equalize for the different physical units and number of data points.





$E_{y|z} = E_y + E_z$

$E_{y|z}=(p/N_y\sigma_y^2)E_y+(q/N_z\sigma_z^2)E_z$

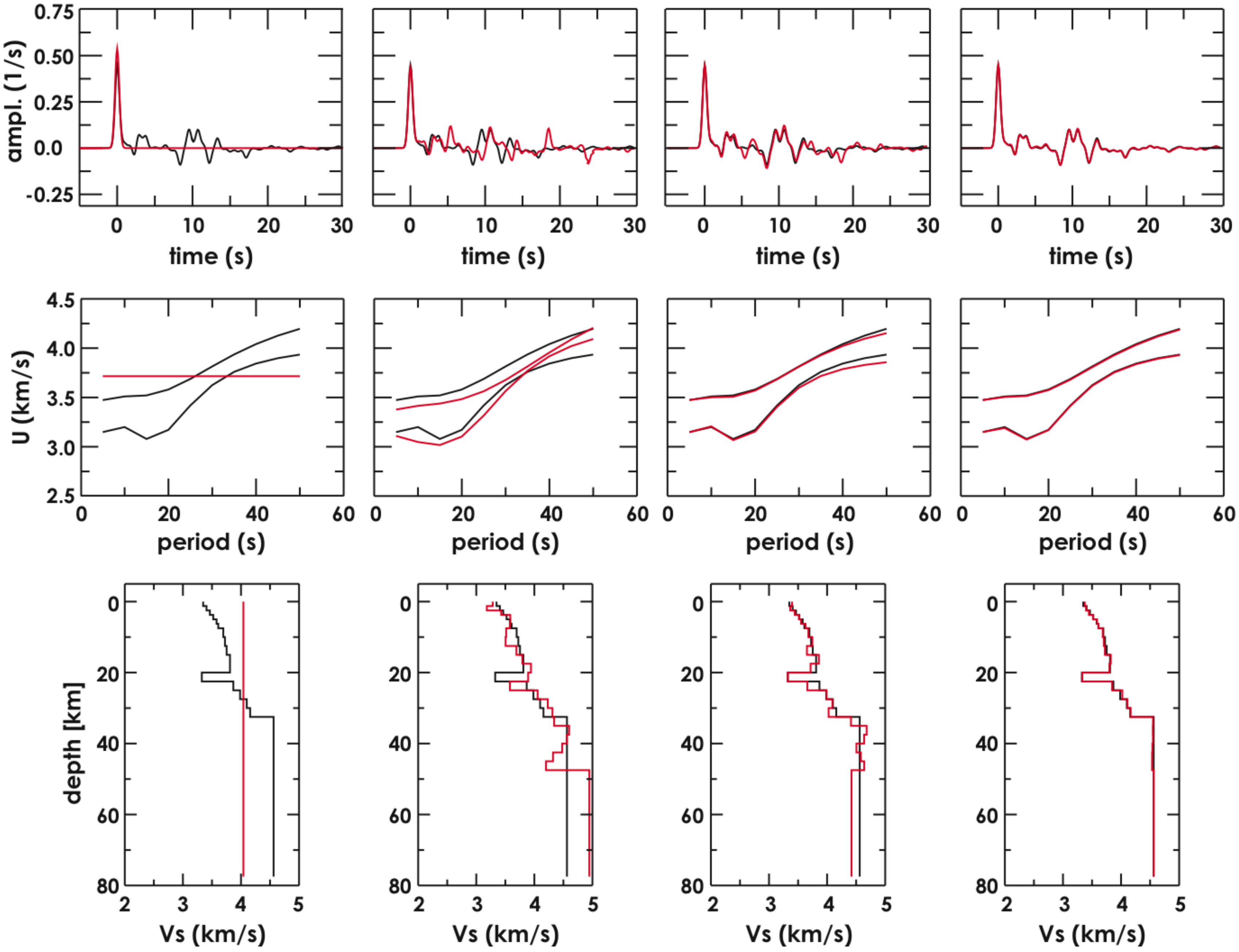
Setting up the joint inversion problem

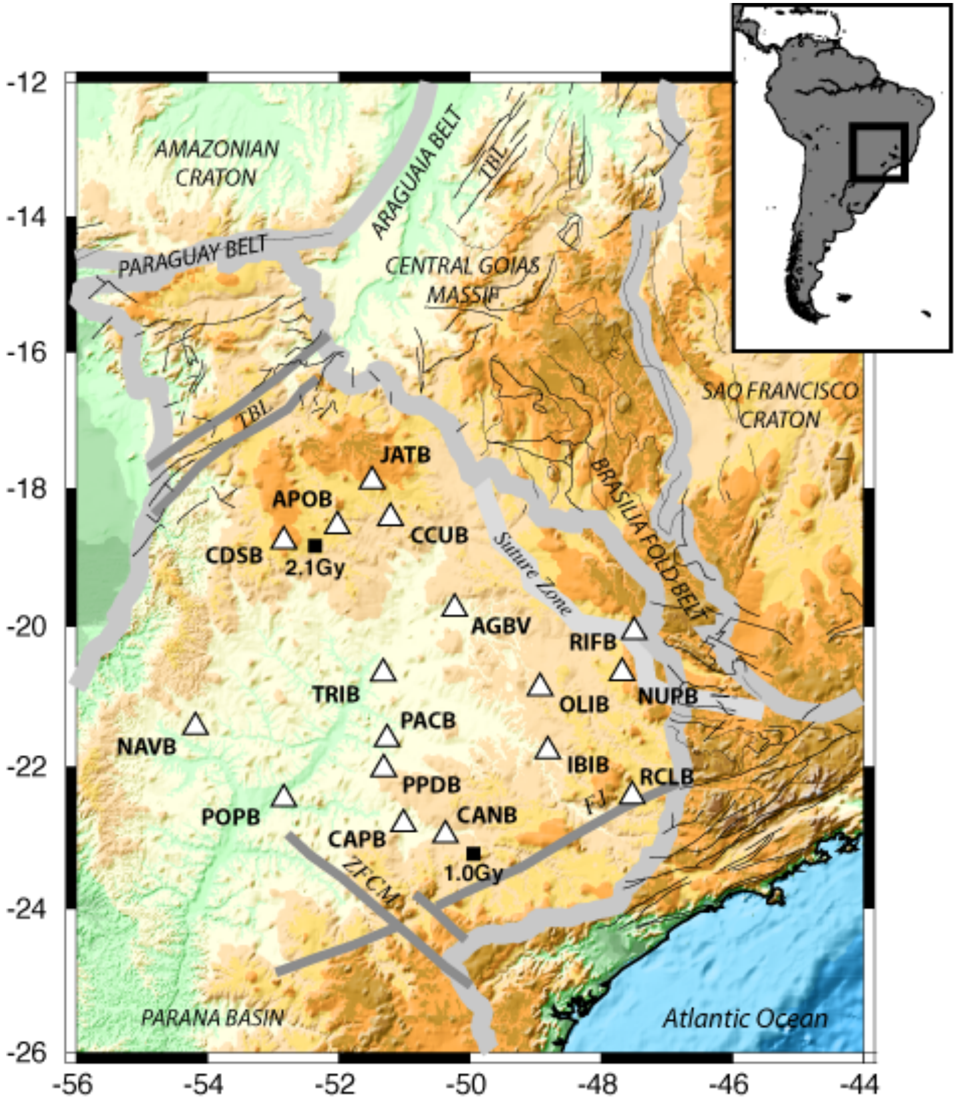
The system of equations that implements the joint inversion is

$$\begin{bmatrix} \sqrt{\frac{p}{w_s^2}}\, D_s \\ \sqrt{\frac{q}{w_b^2}}\, D_b \\ \sigma\Delta \\ A \end{bmatrix} \vec{m} = \begin{bmatrix} \sqrt{\frac{p}{w_s^2}}\, \vec{r}_s \\ \sqrt{\frac{q}{w_b^2}}\, \vec{r}_b \\ \vec{0} \\ A\,\vec{m}_a \end{bmatrix} + \begin{bmatrix} \sqrt{\frac{p}{w_s^2}}\, D_s \\ \sqrt{\frac{q}{w_b^2}}\, D_b \\ \vec{0} \\ \vec{0} \end{bmatrix} \vec{m}_o$$

Where ‘p’ is the so-called influence parameter, q=1-p, and w_x is a normalization factor (taken as $N\sigma^2$).

Δ is the 2nd difference matrix to impose smoothness constraints; and A is a matrix of weights to impose a *priori* constraints $\mathbf{m}_a$ on the inverted velocity model.

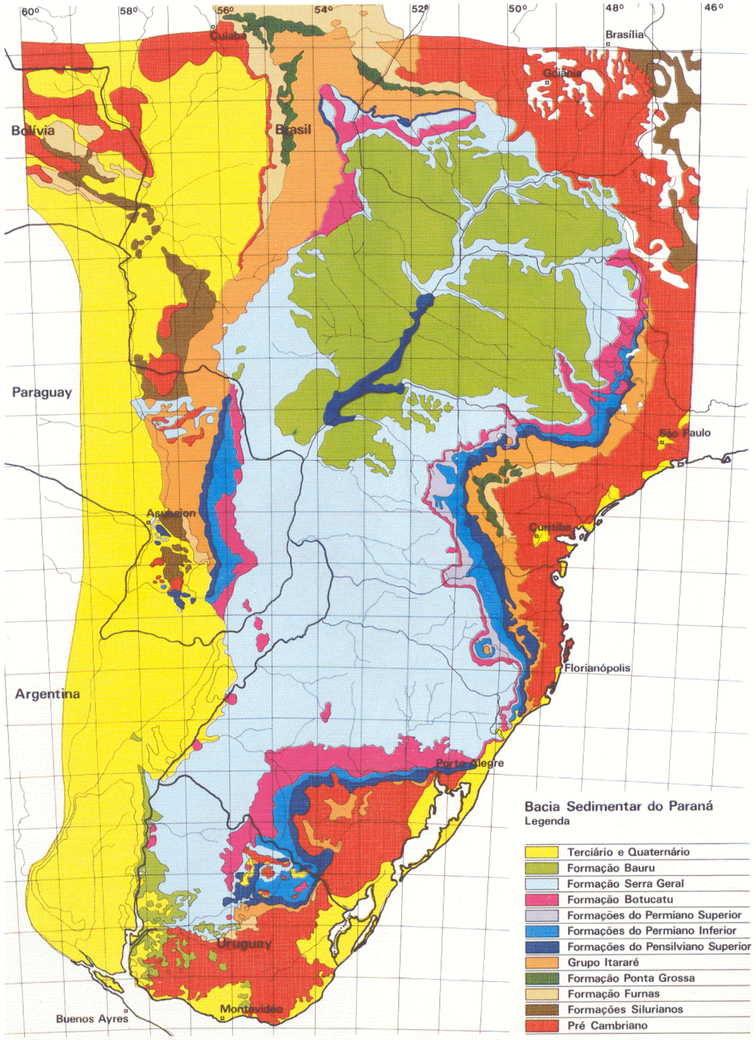

ampl. (1/s)
0.75
0.50
0.25
0.0
-0.25
0
10
20
30
time (s)
time (s)
time (s)
time (s)
U (km/s)
4.5
4.0
3.5
3.0
2.5
0
20
40
60
period (s)
period (s)
period (s)
period (s)
depth [km]
0
20
40
60
80
2
3
4
5
Vs (km/s)
Vs (km/s)
Vs (km/s)
Vs (km/s)



The Paraná Basin of Brazil

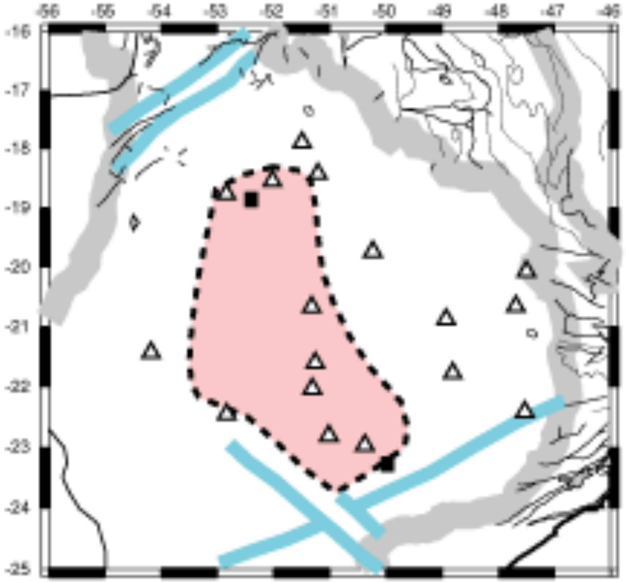
- Initiated during middle to late Ordovician.
- Framed by Proterozoic mobile belts.
- Basement samples date over 2 Ga.
- ~42 km thick crust (including sediments)
- Lower crust ~3.75 km/s.

Also a Large Igneous Province

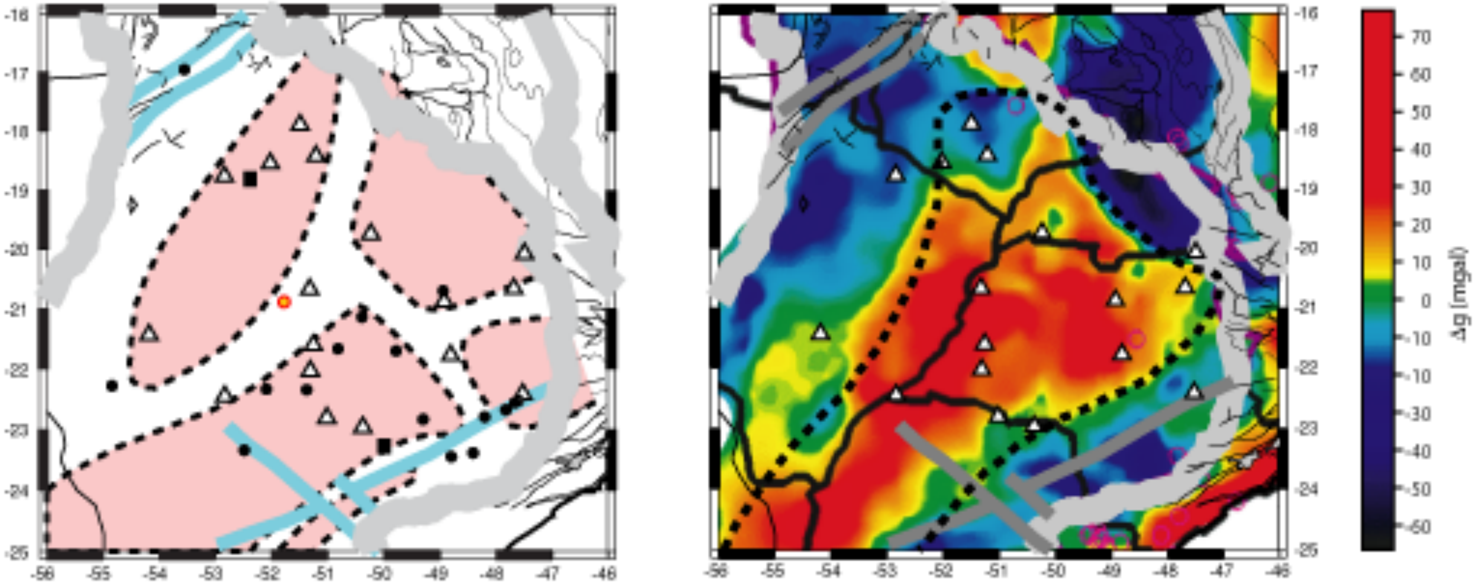


- ~1.5×10^6 km^3 of volcanic rocks in less than 1 My.
- Erupted 137-127 Ma (Cretaceous)
- Mantle plume origin.
- Lack of pervasive mafic underplate suggests a cratonic "root".

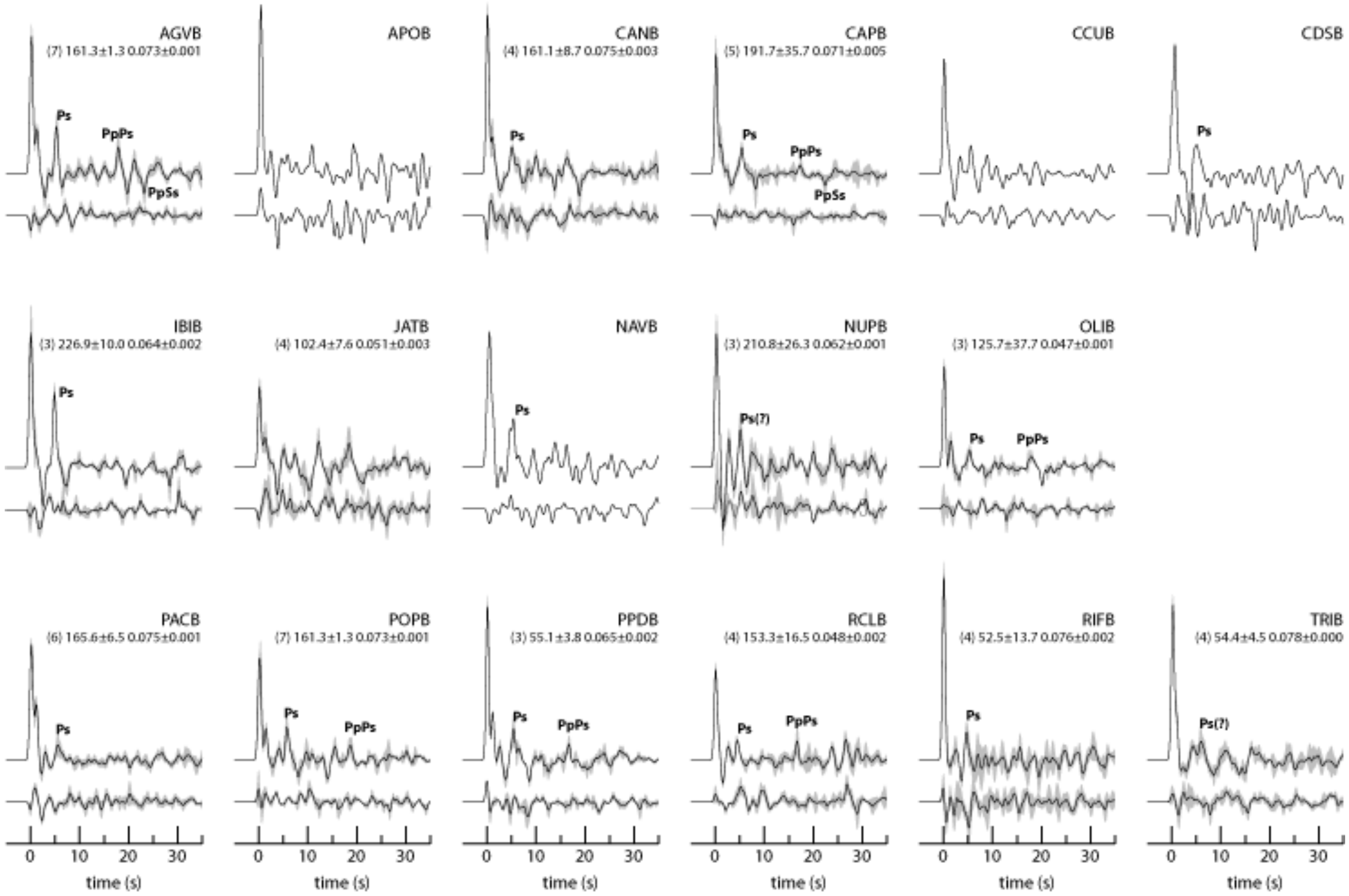
A cratonic nucleus, but …



- SW-NE trending structures from seismic & geophysical surveys.
- *Três Lagoãs* basalts are 443±10 Ma (Neo-Ordovician).

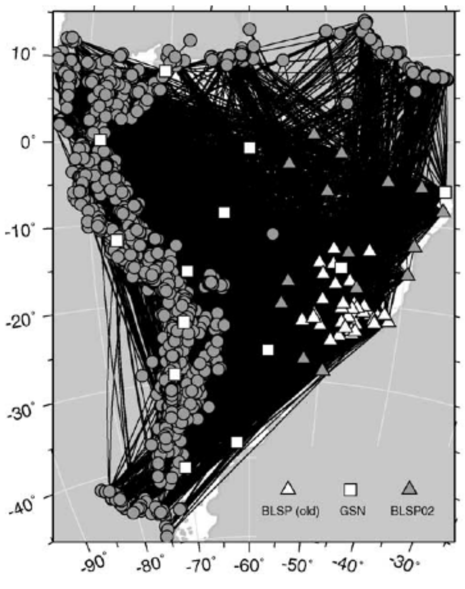


Receiver functions

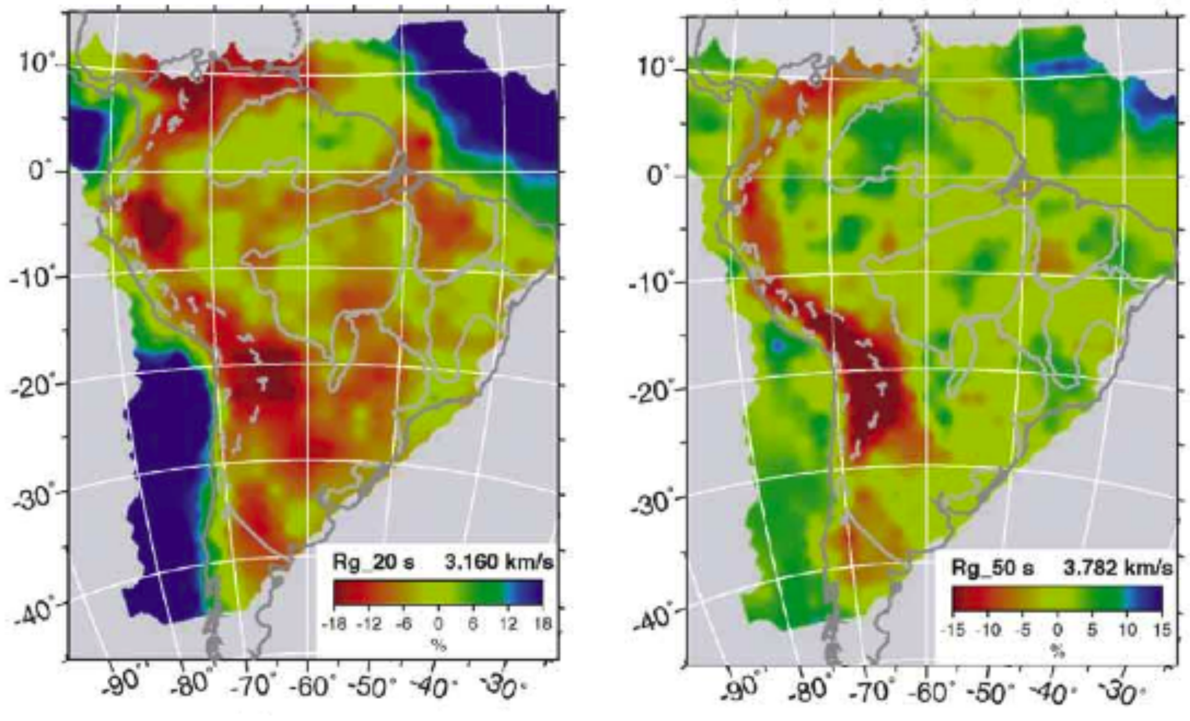


Surface-wave tomography

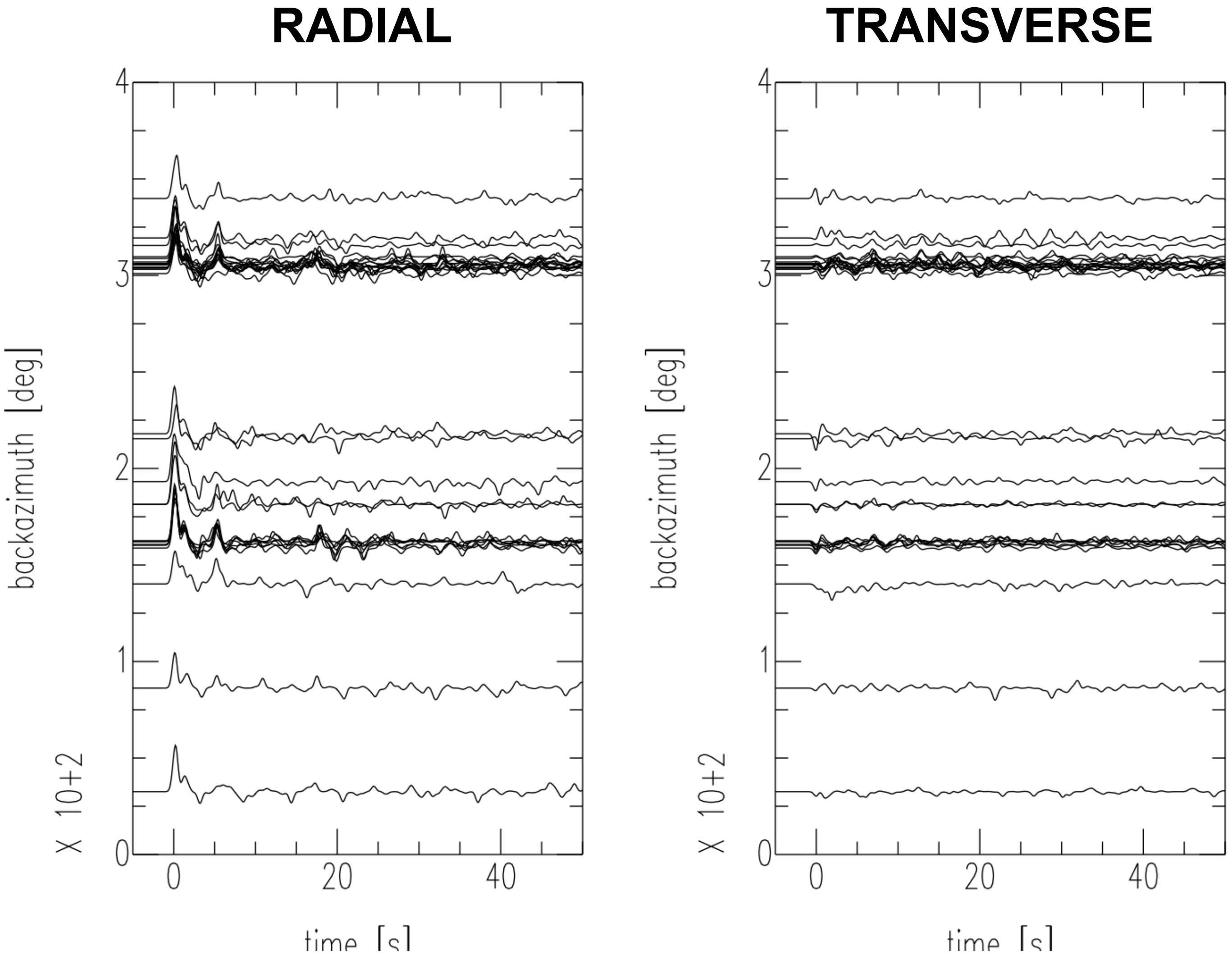
(Feng et al., PEPI, 2004)



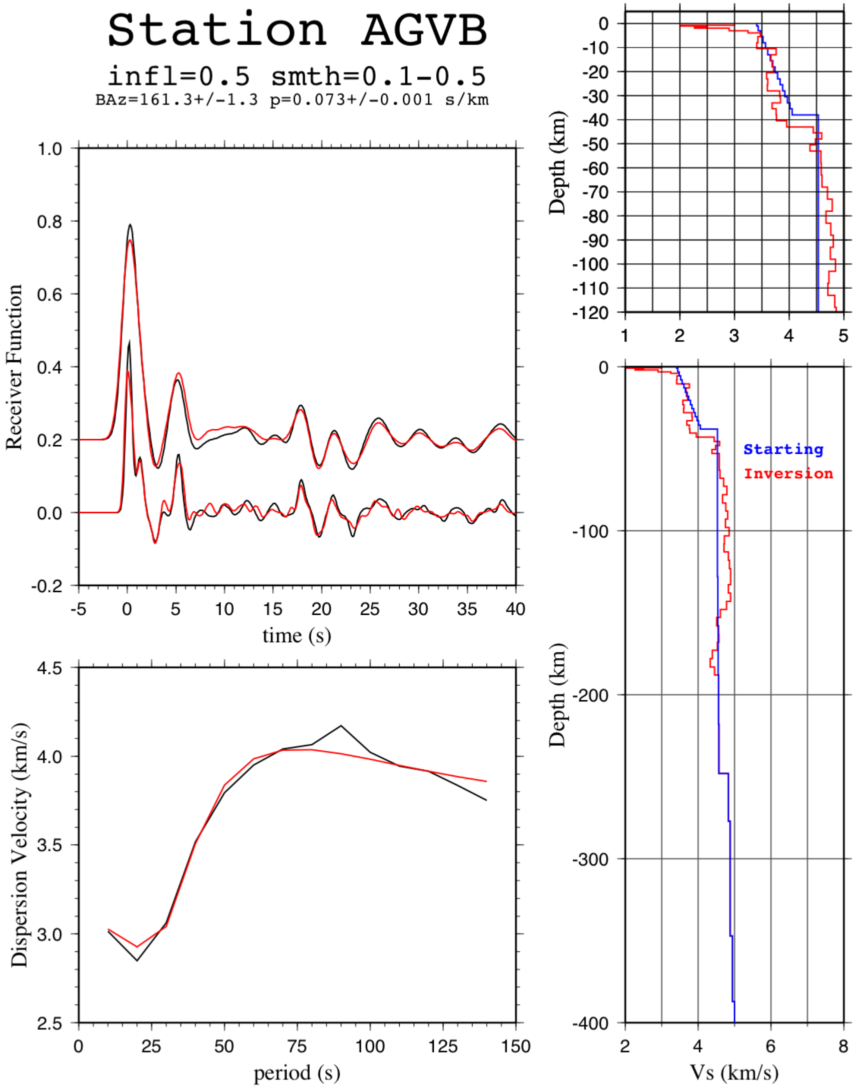
- Group velocities, fundamental mode, Rayleigh wave (10 - 140 s).
- Maximum station density in the Paraná basin (1°x1° cells).



Station AGVB

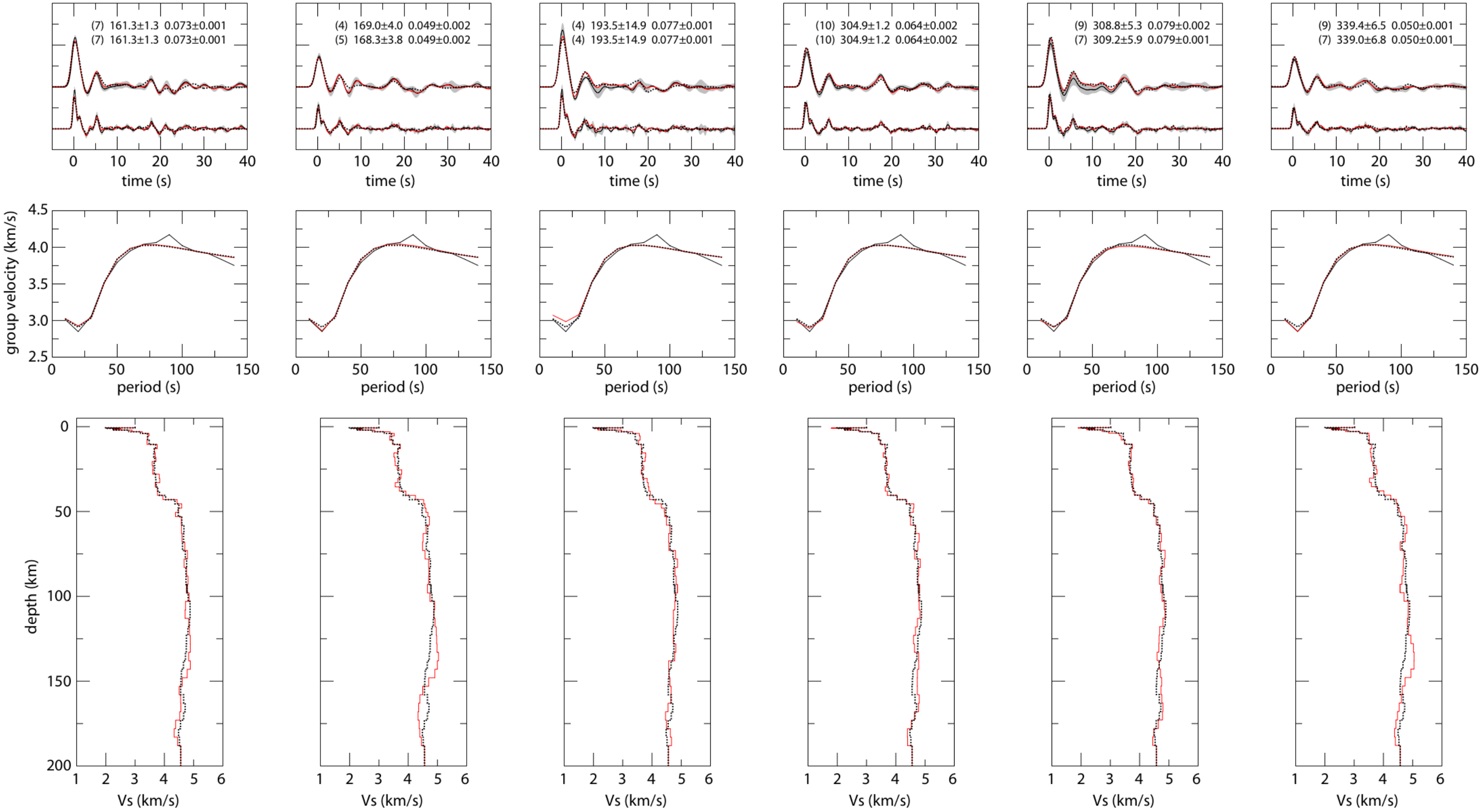


Inversion at station AGVB



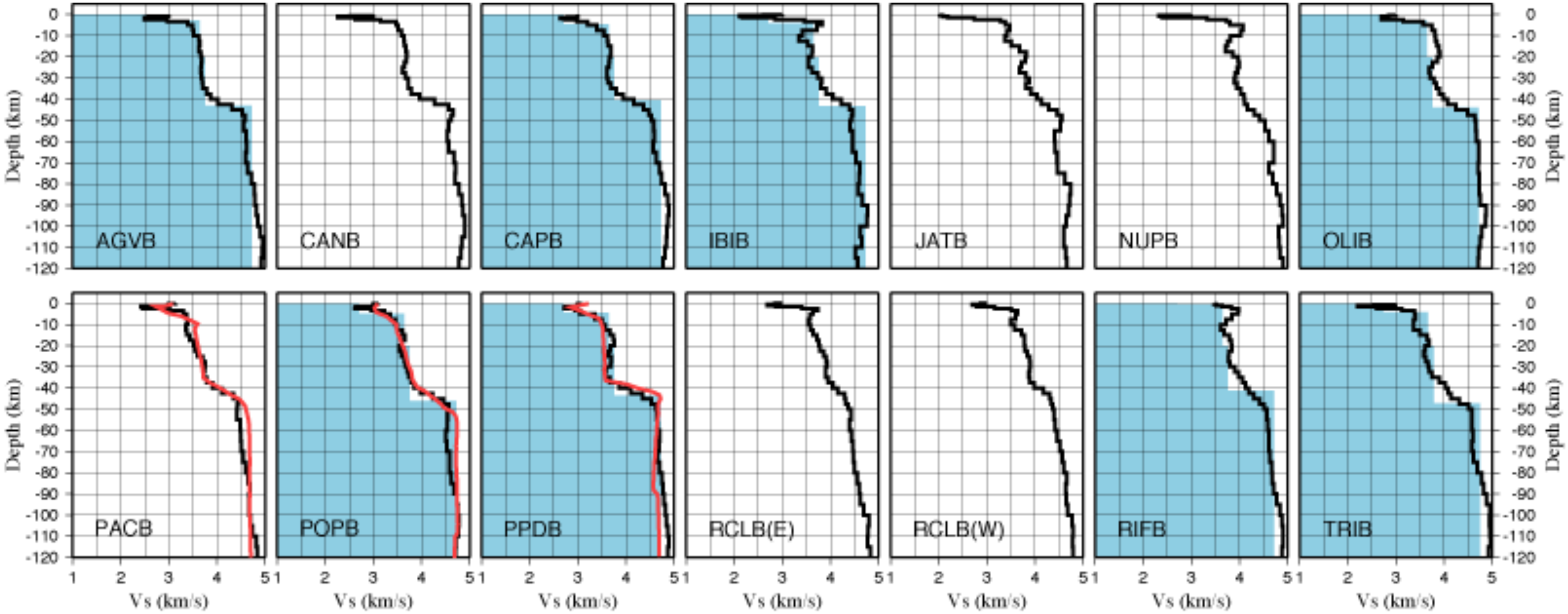
- Receiver functions were obtained in two frequency bands.
- The dispersion curve was borrowed from Feng et al. (2004).
- *A priori* information:
  - Thickness and velocity of the basalt layer.
  - Deeper structure (z > 200 km) is forced to be PREM.
- Smoothness: variable.
- Starting model: gradient over PREM.

Investigating azimuthal dependence



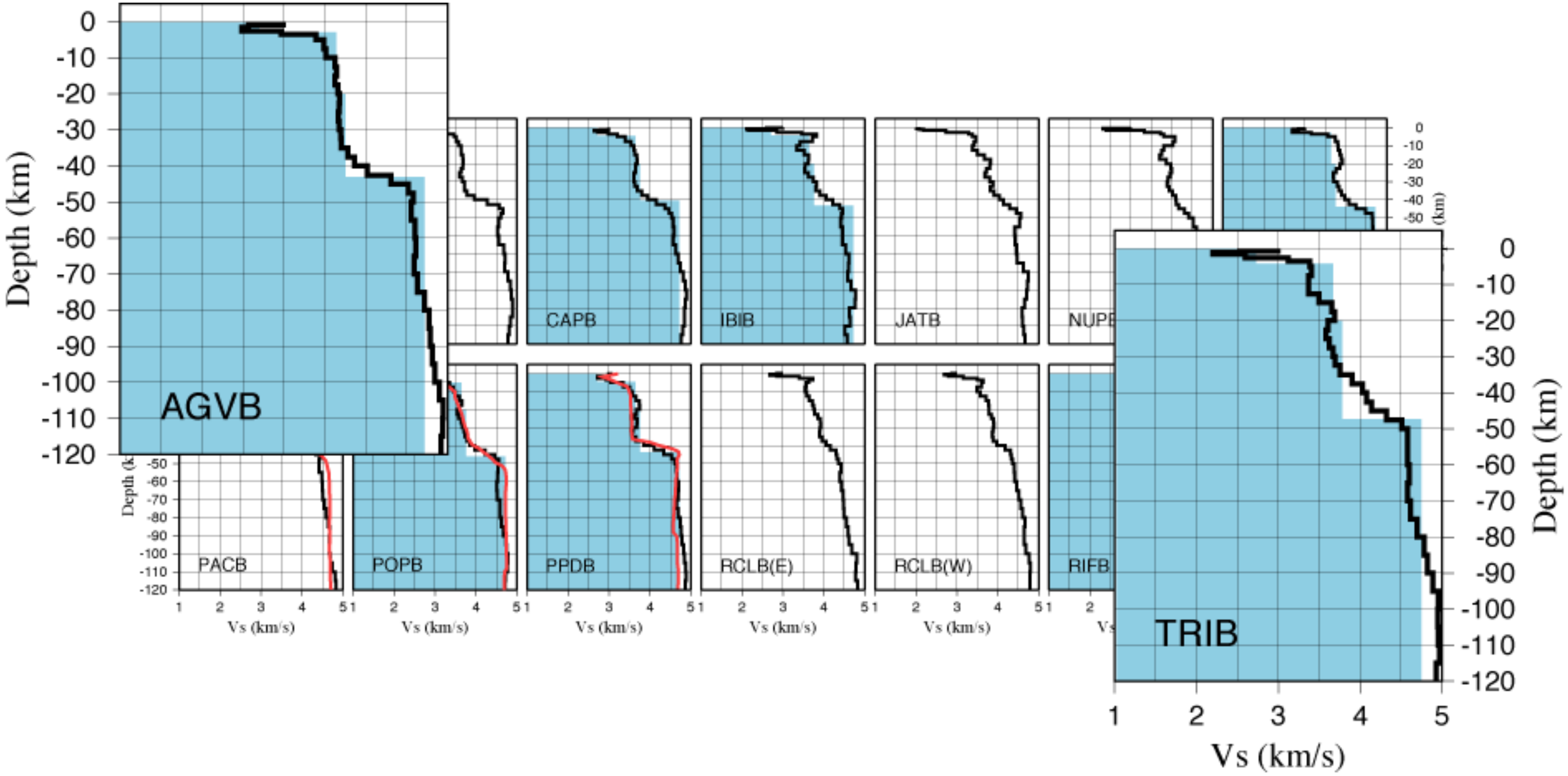
Joint inversion results & comparison

Our joint inversion results are compared to a constrained SW dispersion inversion (Assumpção et al., 1998; blue background) and a joint inversion using inter-station dispersion (An & Assumpção, 2004; red lines).



Joint inversion results & comparison

The comparison reveals 2 types of models: those with a high-speed layer (Vs > 4.0 km/s) above the Moho and those without such a layer.



Correlation with fragmented basement

Stations inside the postulated cratonic blocks do not show a high-speed layer above the Moho. A station well within the suture zones, does display such a layer.